Instituto Português da Qualidade

LABORATÓRIO NACIONAL DE ENGENHARIA CIVIL

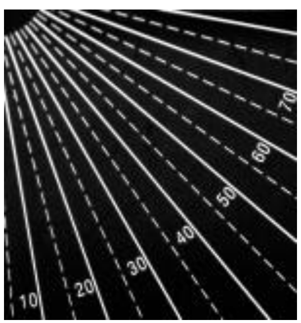
National Physical Laboratory

Quantifying uncertainty in thermal comfort indices

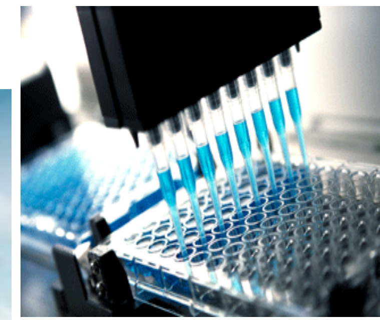
João A Sousa | LNE Workshop EMUE | January 21, 2020

• European Research Projects

The EMPIR initiative is co-funded by the European Union's Horizon 2020 research and innovation programme and the EMPIR Participating States





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MATHMET is the European platform for metrologists, academia industry regarding mathematical and statistical research in all metrological areas by providing

- Guidelines and best-practice guides
- Collaboration in scientific projects
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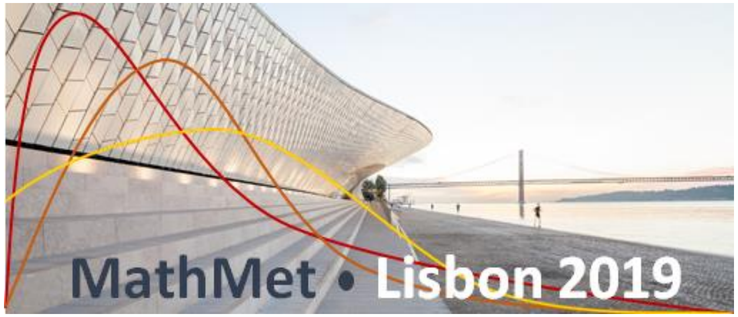

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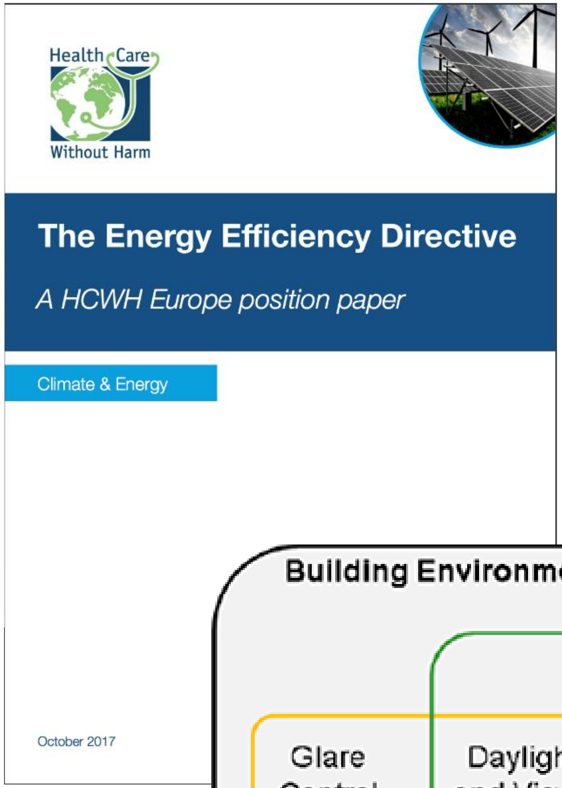
Examples of Measurement Uncertainty Evaluation – EMUE

Objectives

The overall objective is to provide a comprehensive set of new and improved examples to illustrate uncertainty evaluation methods that are in accordance with the GUM and related suite of documents. Some examples will concern the traditional metrology areas of calibration, testing, comparison and conformance evaluation. Selected examples will relate to the thematic areas of environment, energy, quality of life, and industry and society. The examples will be offered to the JCGM and its member organisations (BIPM, IEC, IFCC, ILAC, ISO, IUPAC, IUPAP and OIML) for use in the developing examples document JCGM 110, which will illustrate the application of the GUM suite of documents. They will also be provided to standards committees and other organisations that have expressed a need for them.

Quantifying uncertainty in thermal comfort indices


Health Care Without Harm

The Energy Efficiency Directive

A HCWH Europe position paper

Climate & Energy

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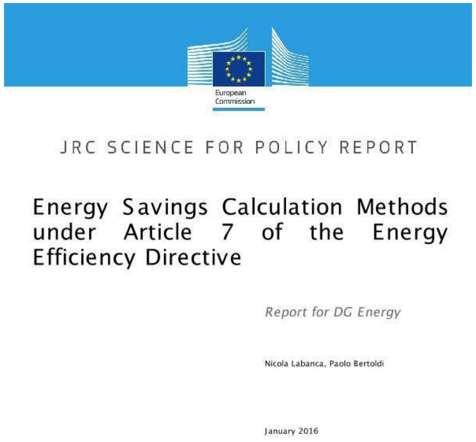
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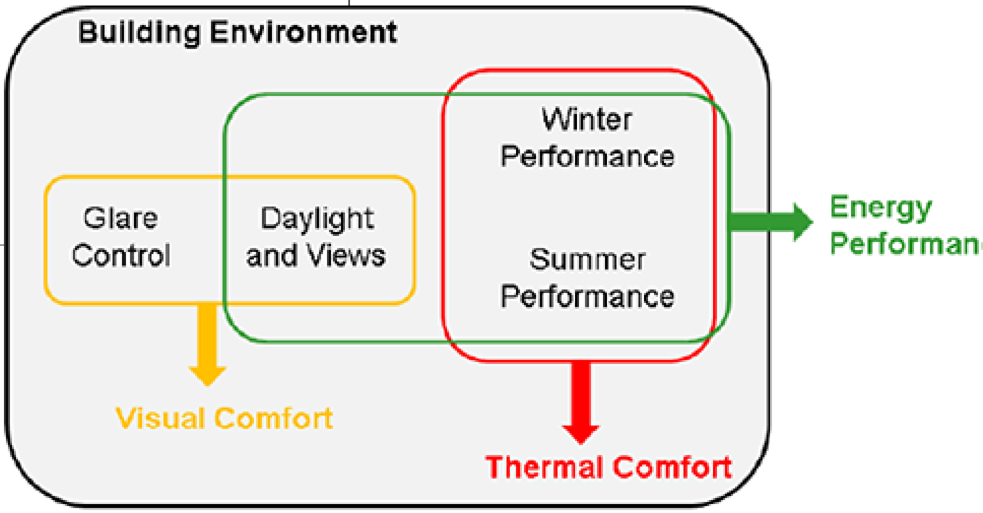
JRC SCIENCE FOR POLICY REPORT

Energy Savings Calculation Methods under Article 7 of the Energy Efficiency Directive

Report for DG Energy

Nicola Labanca, Paolo Bertoldi

January 2016



Quantifying uncertainty in thermal comfort indices

Human comfort inside buildings

Environmental factors

-Space per person
-Level of noise
-Air quality
-Illumination
-Decoration
-Thermal factors

t_a –Air temperature

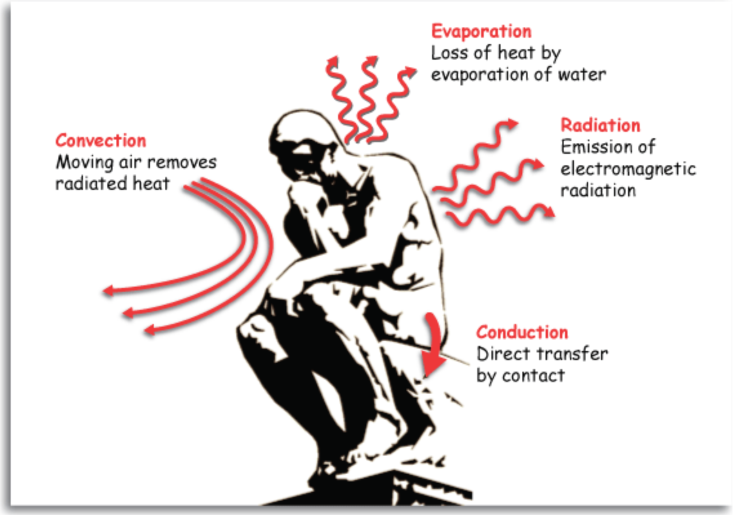
f_a – Relative humidity

v_a –Air velocity

t_r- -Mean radiant temp

Personal factors

-Physical conditions
-Gender
-Age
-Habits
-Clothes
-Activity



Quantifying uncertainty in thermal comfort indices

Thermal comfort perception
+3 hot; +2; +1; 0 neutral; -1; -2; -3 cold

Predicted mean vote (PMV)

$$PMV = \left[0{,}303 \cdot \exp(-0{,}036 \cdot M) + 0{,}028\right] \cdot$$

$$\left\{\begin{aligned} &(M-W) - 3{,}05 \cdot 10^{-3} \cdot \left[5\,733 - 6{,}99 \cdot (M-W) - p_a\right] - 0{,}42 \cdot \left[(M-W) - 58{,}15\right] \\ &-1{,}7 \cdot 10^{-5} \cdot M \cdot (5\,867 - p_a) - 0{,}0014 \cdot M \cdot (34 - t_a) \\ &-3{,}96 \cdot 10^{-8} \cdot f_{cl} \cdot \left[(t_{cl} + 273)^4 - (\bar{t}_r + 273)^4\right] - f_{cl} \cdot h_c \cdot (t_{cl} - t_a) \end{aligned}\right\}$$

where

- M is the metabolic rate, in watts per square metre (W/m^2);
- W is the effective mechanical power, in watts per square metre (W/m^2);
- f_{cl} is the clothing surface area factor;
- t_a is the air temperature, in degrees Celsius (°C);
- $\bar{t}_r$ is the mean radiant temperature, in degrees Celsius (°C);
- p_a is the water vapour partial pressure, in pascals (Pa);
- h_c is the convective heat transfer coefficient, in watts per square metre kelvin [W/(m^2 · K)];
- t_{cl} is the clothing surface temperature, in degrees Celsius (°C).

NOTE 1 metabolic unit = 1 met = 58,2 W/m^2; 1 clothing unit = 1 clo = 0,155 m^2 · °C/W.

Quantifying uncertainty in thermal comfort indices

Mathematical models for thermal comfort indexes

$$
\begin{aligned}
\mathrm{PMV} = {} & \left[0.303\exp(-0.036M)+0.028\right] \\
& \times\Big\{(M-W)-3.05\times10^{-3}\left[5733-6.99(M-W)-p_{\mathrm{a}}\right] \\
& -0.42(M-W-58.15)-1.7\times10^{-5}(5867-p_{\mathrm{a}})M-0.0014(34-t_{\mathrm{a}})M \\
& -\boxed{3.96\times10^{-8}\left[(t_{\mathrm{cl}}+273)^4-(\bar{t}_{\mathrm{r}}+273)^4\right]f_{\mathrm{cl}}-(t_{\mathrm{cl}}-t_{\mathrm{a}})f_{\mathrm{cl}}h_{\mathrm{c}}(t_{\mathrm{cl}})}\Big\},
\end{aligned}
$$

$$
\begin{aligned}
t_{\mathrm{cl}} = {} & 35.7-0.028(M-W) \\
& -I_{\mathrm{cl}}\Big\{\boxed{3.96\times10^{-8}\left[(t_{\mathrm{cl}}+273)^4-(\bar{t}_{\mathrm{r}}+273)^4\right]f_{\mathrm{cl}}+(t_{\mathrm{cl}}-t_{\mathrm{a}})f_{\mathrm{cl}}h_{\mathrm{c}}(t_{\mathrm{cl}})}\Big\},
\end{aligned}
$$

$$
h_{\mathrm{c}}(t_{\mathrm{cl}}) = \begin{cases} 2.38\left|t_{\mathrm{cl}}-t_{\mathrm{a}}\right|^{1/4}, & 2.38\left|t_{\mathrm{cl}}-t_{\mathrm{a}}\right|^{1/4} > 12.1\sqrt{v_{\mathrm{ar}}}, \\ 12.1\sqrt{v_{\mathrm{ar}}}, & 2.38\left|t_{\mathrm{cl}}-t_{\mathrm{a}}\right|^{1/4} \geq 12.1\sqrt{v_{\mathrm{ar}}}, \end{cases}
$$

$$
f_{\mathrm{cl}} = \begin{cases} 1.00+1.290\,I_{\mathrm{cl}}, & I_{\mathrm{cl}} \leq 0.078\,\mathrm{m^2KW^{-1}}, \\ 1.05+0.645\,I_{\mathrm{cl}}, & I_{\mathrm{cl}} > 0.078\,\mathrm{m^2KW^{-1}}, \end{cases}
$$

$$
p_{\mathrm{a}} = \mathrm{RH}\times p_{\mathrm{s}}(t_{\mathrm{a}})10\ \mathrm{Pa}
$$

$$
p_s(t_a) = \exp\left(16.6536-\frac{4030.183}{t_{\mathrm{a}}+235}\right)\mathrm{kPa},
$$

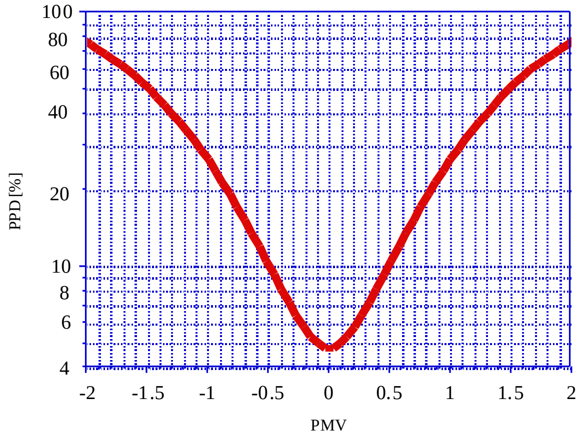
Quantifying uncertainty in thermal comfort indices

Mathematical models for thermal comfort indexes

- The main complication in evaluating PMV for given values of input quantities is that one of these quantities, t_{cl}, is defined implicitly, with its value obtained by iteration.

- Knowing the PMV index, the PPD index predicts the percentage of thermally dissatisfied persons feeling uncomfortable (± 3 and ± 2 votes on the thermal sensation scale). The relation between the two parameters is shown in the figure below

$$\mathrm{PPD} = 100 - 95\exp\left[-0.03353(\mathrm{PMV})^4 - 0.2179(\mathrm{PMV})^2\right].$$



Quantifying uncertainty in thermal comfort indices

Continuous and discrete scales

- PMV is defined on a *continuous* scale. However it is interpreted on a *discrete* scale. By rounding PMV to the nearest integer, PMV is defined on a 7-point thermal sensation scale:

-3/cold, -2/cool, -1/slightly cool, 0/neutral, +1/slightly warm, +2/warm, +3/hot

- ISO 7730:2005 states the index should only be used for values of PMV between -2 and +2 and when the quantities that influence it lie in stipulated intervals. It also states that PMV can be used to check whether a given thermal environment complies with comfort criteria
- Considering $\mathrm{RPMV} = \min\{\max[-3, \mathrm{round}(\mathrm{PMV})], 3\}$

the uncertainty $u(\widehat{\mathrm{RPMV}})$ associated with a particular value $\widehat{\mathrm{RPMV}}$ of RPMV is given by

$$u^2(\widehat{\mathrm{RPMV}}) = u^2(\widehat{\mathrm{PMV}}) + (\widehat{\mathrm{RPMV}} - \widehat{\mathrm{PMV}})^2$$

which can lead to $|\widehat{\mathrm{RPMV}} - \widehat{\mathrm{PMV}}| \approx 0.5$ and $u^2 = (\widehat{\mathrm{RPMV}})$ exceeding $u^2 = (\widehat{\mathrm{PMV}})$ by approximately 0.25 which would be totally artificial

Quantifying uncertainty in thermal comfort indices

Approach to uncertainty evaluation

GUM uncertainty framework

- Linear mathematical model (exact solution using LPU)
- Differentiable mathematical model
- All input quantities have symmetrical PDF centred at zero
- Output quantity has a Gaussian PDF (symmetrical and centred at its mean value)
- Assumes valid conditions for the central limit theorem
- Uses the Welch-Satterthwaite expression to determine the number of degrees of freedom
- LPU only an approximation for other all cases
- Multivariable models are not covered by this approach

Required input

These are obtained from PDFs that capture knowledge of the quantities concerned

- Estimates $\boldsymbol{x}$ of the input quantities $\boldsymbol{X}$
- Standard uncertainties $u(x_i)$ associated with the x_i
- Covariances $\mathrm{cov}(x_i, x_j)$ associated with x_i and x_j

Quantifying uncertainty in thermal comfort indices

Approach to uncertainty evaluation

GUM uncertainty framework

Explicit model

- The estimate of the output quantity is taken as $y = f(\boldsymbol{x})$. Defining the covariance matrix

$$\boldsymbol{U}_x = \begin{bmatrix} u(x_1, x_1) & \cdots & u(x_1, x_N) \\ \vdots & \ddots & \vdots \\ u(x_N, x_1) & \cdots & u(x_N, x_N) \end{bmatrix}$$

containing the covariances $u(x_i, x_j)$, and the (row) vector $\boldsymbol{c}^{\mathrm{T}} = [c_1, \cdots, c_N]$ containing the sensitivity coefficients, then the standard uncertainty associated with y is evaluated from

$$u^2(y) = \boldsymbol{c}^{\mathrm{T}} \boldsymbol{U}_x \boldsymbol{c}$$

- For independent input quantities, the variance can be seen as a sum of terms

$$u^2(y) = \sum_{i=1}^{N} [c_i u(x_i)]^2 = \sum_{i=1}^{N} u_i^2(y), \qquad u_i(y) \equiv |c_i| u(x_i)$$

Quantifying uncertainty in thermal comfort indices

Approach to uncertainty evaluation

Implicit model

- In an univariate real implicit measurement model, a single real output quantity Y is related to real input quantities X in a way that cannot readily be represented in terms of a direct functional relationship. Such model takes the general form

$$h(Y, \boldsymbol{X}) = 0$$

The estimate y of Y is the value of η that solves the equation $h(\eta, x) = 0$. This equation is solved numerically for y using a suitable zero-finding algorithm.

- The standard uncertainty $u(y)$ associated with y is evaluated from

$$u^2(y)\, c_y^2 = \boldsymbol{c}_x^{\mathrm{T}} \boldsymbol{U}_x \boldsymbol{c}$$

where $\boldsymbol{c}_x^{\mathrm{T}}$ is the (row) vector of dimensional $1 \times N$ of partial derivatives $\partial h/\partial Xi$, and c_y is the partial derivative $\partial h/\partial Y$, with all derivatives evaluated at $\boldsymbol{X} = \boldsymbol{x}$ and $Y = y$.

Quantifying uncertainty in thermal comfort indices

Approach to uncertainty evaluation

Propagation of distribution – Monte Carlo method

- PDFs for the input quantities are propagated through the measurement model to provide the PDF for the output quantity
- Expectation of output PDF is used as the best estimate of the measurand
- Standard deviation of output PDF is used as the standard deviation associated with the measurand
- Monte Carlo methods should be used when the applicability of the GUM uncertainty framework is questionable. Makes no linearizing or shape assumptions
- Numerical accuracy needs to be checked
- Can be used to validate GUM uncertainty framework
- Once the PDF for the output quantity is available, a coverage interval for Y corresponding to any particular coverage probability p can be obtained

Quantifying uncertainty in thermal comfort indices

Input data

Tabulated values

- Values from ISO 7730:2005 would be taken as having a rectangular distribution, to account for their uncertainty, but in this instance we decided to used them as fixed values. It is a relevant factor.
- Metabolic rate with resolution $1\ \mathrm{Wm}^{-2} \Rightarrow (b-a)/\sqrt{12} = 0.3\ \mathrm{Wm}^{-2}$
- Air temperature with resolution $0.5\ \mathrm{ºC} \Rightarrow (b-a)/\sqrt{12} = 0.14\ \mathrm{ºC}$
- Air temperature with resolution $0.1\ \mathrm{ºC} \Rightarrow (b-a)/\sqrt{12} = 0.03\ \mathrm{ºC}$

Measured values

- 40 readings in different places of elderly homes
- Common instruments
- Standard uncertainty based on repeatability and accuracy of instruments, so that $u = (u_{\mathrm{A}}^2 + u_{\mathrm{B}}^2)^2$

Quantifying uncertainty in thermal comfort indices

Input data

- Input quantities used in the mathematical models

Table 3 Best estimate of the input quantities

	M /met	W /Wm^{-2}	I_{cl} /clo	t_a /ºC	t_r /ºC	RH /%	v_{ar} /ms^{-1}
Run #1	1.2	0	0.5	22.0	22.0	60	0.1
Run #2	1.2	0	0.5	27.0	27.0	60	0.3
Run #3	1.2	0	0.5	23.5	25.5	60	0.3
Run #4	1.2	0	1.0	23.5	23.5	40	0.3
Run #5	1.6	0	0.5	27.0	27.0	60	0.1

Table 4 Standard uncertainties associated with the best estimate of the measured quantities in Table 3

Location	Type	t_a /ºC	t_r /ºC	RH /%	v_{ar} /ms^{-1}
Office 1	A	0.0	0.0	0.1	0.00
Office 2	A	0.0	0.0	0.0	0.00
Customer service	A	0.0	0.0	0.0	0.01
All	B	0.1	0.2	0.5	0.05

Results

- GUM uncertainty framework

Table 5 GUM uncertainty budget for the PMV model (neg = negligible)

Quantity	PDF	Best estimate	Standard uncertainty	c_i	u_i (PMV)
M /Wm^{-2}	Ref. value	70	-	-	-
W /Wm^{-2}	Ref. value	0	-	-	-
t_a /ºC	Combined	22.0	0.1	0.028	0.013
I_{cl} / m^{2}KW^{-1}	Ref. value	0.078	-	-	-
t_r /ºC	Gaussian	22.0	neg	-	-
RH /%	Rectangular	60.0	0.3	0.0059	0.0017
v_{ar} /ms^{-1}	Rectangular	0.10	0.03	3.27	0.094
PMV		–0.75	u(PMV)=0.094	0.5	$U_{0.95}$(PMV)=0.18

- Air velocity, in this case, is the dominant factor on the perception of thermal comfort (expressed by PMV). In this model its influence is through h_c. The PDF of h_c is very sensitive to velocity with similar shape of the output PMV. Sensitivity analysis on PMV model showed same results.

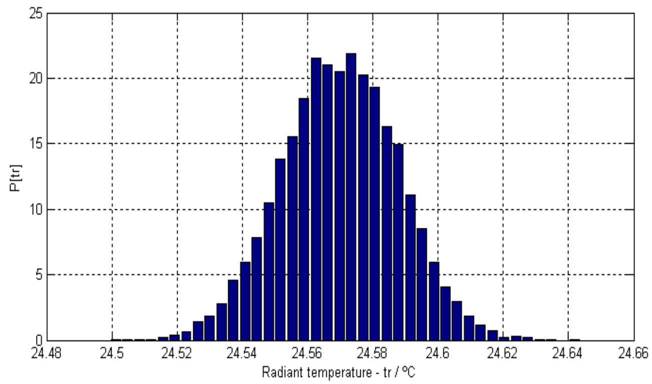
Quantifying uncertainty in thermal comfort indices

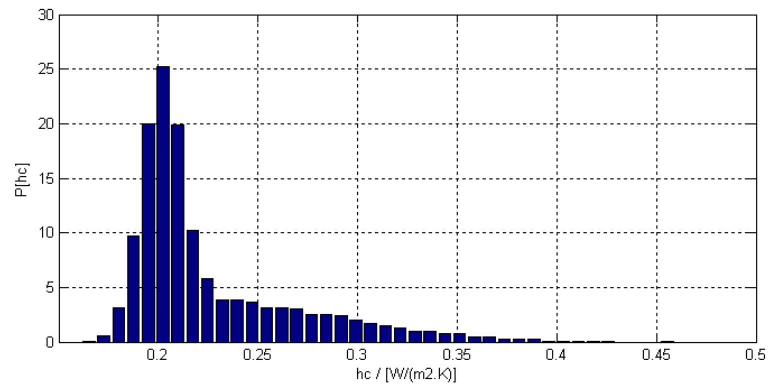
Results

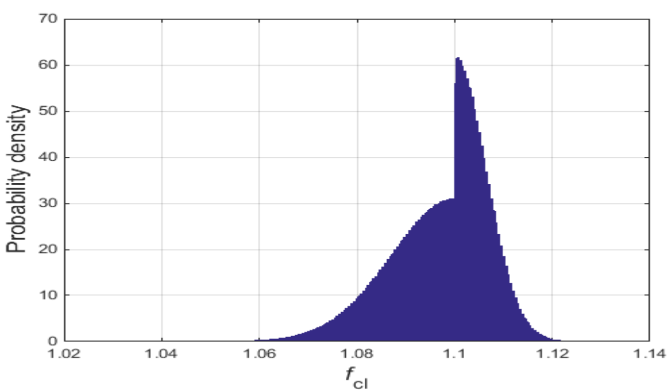
GUM uncertainty framework ⇒
All input quantities have symmetrical PDF centred at zero

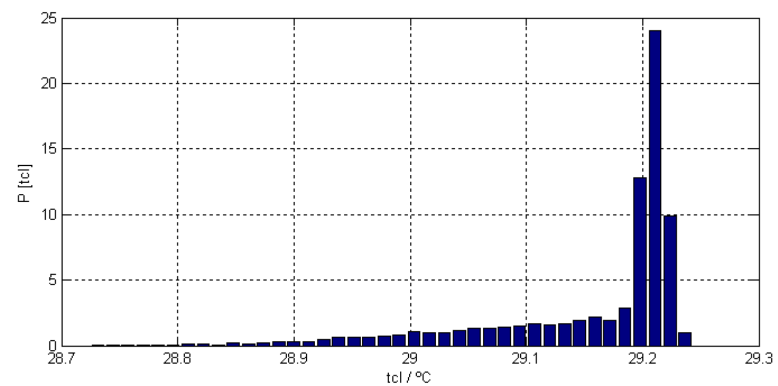
Input quantities

- PDFs of some input quantities









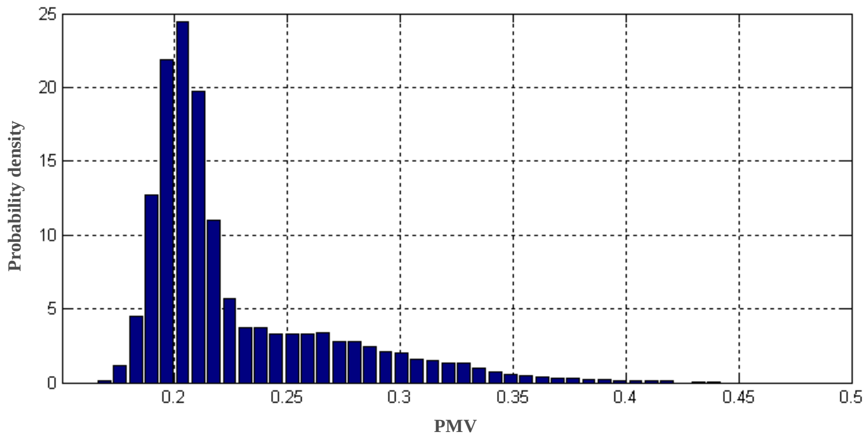
Quantifying uncertainty in thermal comfort indices

Results

GUM uncertainty framework ⇒
Assumes valid conditions for the central limit theorem

PMV index

- PDFs of thermal comfort indexes



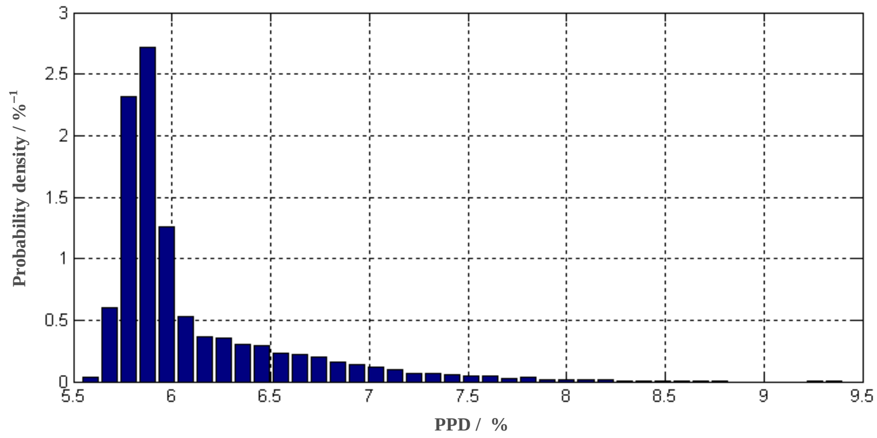
- Advantages of MC over GUM are apparent in this application. GUM only delivers best estimate and standard uncertainty, MC gives insight, providing much richer information, such as knowledge on tails of the PDF for the measurands. Long tail probably means high value for PMV index when compared with GUM application

Quantifying uncertainty in thermal comfort indices

Results

PPD index and results

- Same conclusions as for the PPD index. The expression for this index reduces approximately to a quadratic in PMV of the form $5 + C(\mathrm{PMV})^2$ for some positive constant C, for the data use. Not surprisingly both PDFs are similar.



Quantity	Best estimate	Standard uncertainty	95 % coverage interval		
			Lower limit	Upper limit	Length
PMV	0.23	0.04	0.06	0.24	0.18
PPD /%	5.5	0.3	5.1	6.2	1.1

Quantifying uncertainty in thermal comfort indices

Results

Sensitivity analysis

- A study was carried out to provide a sensitivity analysis for the input parameters. Small variations were introduced successively to approximate the partial derivatives
- For the quantities obtained by measurement, results showed linear behaviour with air velocity having the greatest influence, due to the effect on convection and thus on the clothing surface temperature
- For the tabular values (taken commonly as exact values), the analysis showed a nonlinear behaviour in the neighbourhood of the testing points.
- In terms of uncertainty simulation for these quantities, assuming a Gaussian PDF N(1.2, $(0.05)^2$) for M (metabolism) produced a similar result for PMV (0.23) and its expanded uncertainty (0.16)
- As for the I_{cl} quantity, assuming a Gaussian PDF N(0.7, $(0.05)^2$) produces a significant influence on the result, with a different estimate for PMV index (0.07) and an expanded uncertainty of (2.5)

Quantifying uncertainty in thermal comfort indices

Results

Comparison of results

- As a validation study results from the Monte Carlo calculation were compared with results from reference test sets for input quantities with known output quantities (PMV index) according to ISO 7730:2005

Run nº	Air temperature /ºC	Mean radiant temperature /ºC	Air speed / ms^{-1}	RH /%	Metabolic rate met	Clothing insulation clo	PMV (expd)	PMV (MCM)	PPD (expd)	PPD (MCM)
1	22,0	22,0	0,10	60	1,2	0,5	–0,75	–0,76	17	17,4
3	27,0	27,0	0,30	60	1,2	0,5	0,44	0,43	9	9,0
5	23,5	25,5	0,30	60	1,2	0,5	–0,55	–0,56	11	11,6
8	23,5	23,5	0,30	40	1,2	1,0	0,12	0,12	5	5,3
12	27,0	27,0	0,10	60	1,6	0,5	1,17	1,18	34	34,5

- The comparison between results produced by the GUM uncertainty framework and Monte Carlo showed comparable results, especially for the estimates of PMV and PPD

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Quantifying uncertainty in thermal comfort indices

Conclusions

- The Monte Carlo method was found to be particularly suitable to handle the complexity of the mathematical model that specifies the implicit relationship between convection and clothing surface temperature
- The comparison of the measurement uncertainty with the scale intervals led to the conclusion that, within these experimental conditions, uncertainty is very large, about 50 % of the length of the scale interval and thus can not be ignored in any decision making
- The convective heat transfer coefficient h_c produces an asymmetric PDF having significant influence on the shape of the PDFs for PMV and PPD
- The parametric studies carried out showed that the air velocity was the major influential quantity, and also showed that the reported value of the PMV index should take into consideration its associated uncertainty
- The uncertainty of tabulated values should be studied and taken into consideration

Quantifying uncertainty in thermal comfort indices

Scope for future work

- A more thorough investigation on the tabulated values should be attempted. This includes a parametric study involving, e.g., thermal comfort perception for a range of different levels of activity (metabolic rates)
- The values of uncertainties associated with M and I_{cl}, but especially with the latter, should be simulated since preliminary results indicate an important influence that cannot be ignored. More detailed information is required (ISO 9920:2007)
- There are values for air velocities close to zero (small air movement) which raises questions on the use of the GUM and the possible existence of negative values of air velocity. A Bayesian treatment would enable better use of all available information

Merci beaucoup!