The EMPIR initiative is co-funded by the European Union's Horizon 2020 research and innovation programme and the EMPIR Participating States

EMUE
Examples of Measurement Uncertainty Evaluation

Reporting measurement results clearly

Maurice Cox
National Physical Laboratory UK

M18 meeting LNE Paris January 2020

Outline

- Reporting measurement results fundamental since primary outcome of MU evaluation
- Types of reporting
  - ‘Conventional’
  - Uncertainty range
  - Probability distribution
- e-certificates
  - Currently being announced/considered
  - Present opportunities
  - Report summary statistics and probability distribution from which obtained
- International Vocabulary of Metrology (VIM)
- Much information available: GUM, GUM supplements, EA, Eurachem, UKAS, ILAC, ...
- Numerical accuracy/rounding
- Correlated quantities

What is a measurement result?

JCGM 200:2012
International Vocabulary of Metrology – Basic and General Concepts and Associated Terms (VIM 3rd edition)

2.9 (3.1)
measurement result
result of measurement
set of values being attributed to a measurand together with any other available relevant information

What is a measurement result?

JCGM 200:2012
International Vocabulary of Metrology – Basic and General Concepts and Associated Terms (VIM 3rd edition)

2.9 (3.1)
measurement result
result of measurement
set of values being attributed to a measurand together with any other available relevant information

NOTE 1 A measurement result generally contains "relevant information" about the set of quantity values, such that some may be more representative of the measurand than others. This may be expressed in the form of a probability density function (PDF).

NOTE 2 A measurement result is generally expressed as a single measured value and a measurement uncertainty. . . .

. . .

Single measurand: JCGM 100:2008 (GUM)

- Specification of measurand
- Specification of measurement model: $Y = f(X_1, \ldots, X_N)$
- Measure of uncertainty: standard uncertainty
  - Estimate y, associated standard uncertainty $u(y)$
  - Where appropriate, relative standard uncertainty: $u(y)/|y|$, $y \neq 0$

Single measurand: JCGM 100:2008 (GUM)

- Specification of measurand
- Specification of measurement model: $Y = f(X_1, \ldots, X_N)$
- Measure of uncertainty: standard uncertainty
  - Estimate y, associated standard uncertainty $u(y)$
  - Where appropriate, relative standard uncertainty: $u(y)/|y|$, $y \neq 0$
- Measure of uncertainty: expanded uncertainty
  - Measurement result $Y = y \pm U$, where expanded uncertainty $U = ku(y)$
  - Estimate y, associated standard uncertainty $u(y)$
  - Coverage factor k, coverage probability p
  - Where appropriate, relative expanded uncertainty: $U/|y|$, $y \neq 0$
  - Limitation: symmetric probability distribution; later

Multivariate measurands: JCGM 102:2011

- Specification of measurand
- Specification of measurement model: $\boldsymbol{Y} = \boldsymbol{f}(X_1, \ldots, X_N)$
- Either

    estimate $\boldsymbol{y}$, covariance matrix $\boldsymbol{V_x}$

  or

    estimate $\boldsymbol{y}$, vector of standard uncertainties $\boldsymbol{u}(\boldsymbol{y})$, correlation matrix $\boldsymbol{R_x}$

Multivariate measurands: JCGM 102:2011

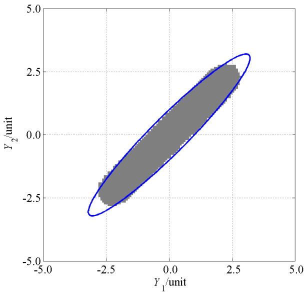
- Specification of measurand
- Specification of measurement model: $\boldsymbol{Y} = \boldsymbol{f}(X_1, \ldots, X_N)$
- Either

  estimate $\boldsymbol{y}$, covariance matrix $\boldsymbol{V_x}$

  or

  estimate $\boldsymbol{y}$, vector of standard uncertainties $\boldsymbol{u}(\boldsymbol{y})$, correlation matrix $\boldsymbol{R_x}$

- Coverage regions
  - Rectangular
  - Ellipsoidal
  - Smallest
- Limitations!



Covariance matrix or correlation matrix?

Gas analysis example from ISO/TS 28038:2018 (polynomial calibration)

Least-squares quadratic model in Chebyshev polynomial form

Covariance matrix for Chebyshev coefficients

$$\boldsymbol{V_y} = 10^{-7} \times \begin{bmatrix} 6.1 & 7.2 & 5.3 \\ & 10.8 & 4.9 \\ \text{sym.} & & 7.4 \end{bmatrix}$$

Covariance matrix or correlation matrix?

Gas analysis example from ISO/TS 28038:2018 (polynomial calibration)

Least-squares quadratic model in Chebyshev polynomial form

Covariance matrix for Chebyshev coefficients

$$\boldsymbol{V_y} = 10^{-7} \times \begin{bmatrix} 6.1 & 7.2 & 5.3 \\ & 10.8 & 4.9 \\ \text{sym.} & & 7.4 \end{bmatrix}$$

Standard uncertainties and correlation matrix

$$\boldsymbol{u}(\boldsymbol{y}) = \begin{bmatrix} 0.0008 \\ 0.0010 \\ 0.0009 \end{bmatrix}, \qquad \boldsymbol{R_y} = \begin{bmatrix} 1 & 0.89 & 0.79 \\ & 1 & 0.54 \\ \text{sym.} & & 1 \end{bmatrix}$$

$$\boldsymbol{V_y} = \boldsymbol{D}\boldsymbol{R_y}\boldsymbol{D}, \qquad \boldsymbol{D} = \mathrm{diag}(\boldsymbol{V_y})$$

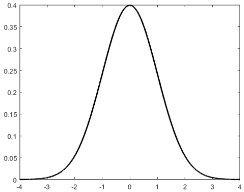
CMC uncertainties

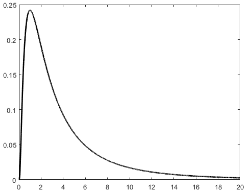
- Four classes of CMC uncertainty claims relate to (a) individual values, (b) intervals (ranges) of values, (c) 2- and 3-dimensional arrays of values, (d) mathematical formulæ
- Commonest class, (b): laboratories provide uncertainty claims for a measurand that depends continuously on a parameter (frequency, wavelength, mass concentration, ...) having an range of values
- In such cases, CMC uncertainty claim is a function of the parameter
- Length measurement example: $(a + bL^2)^{1/2}$
- Algebraic rather than purely numerical representation of uncertainty

Uncertainty statements derived from PDFs

- Monte Carlo (JCGM 101, JCGM 102) and Bayesian calculations yield a PDF for the measurand

- Report
  - Summary statistics,
  - PDF, or
  - Both

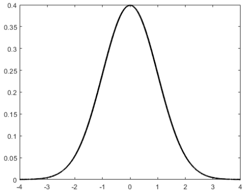


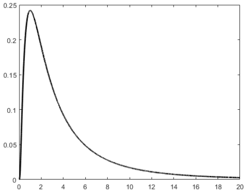


Uncertainty statements derived from PDFs

- Monte Carlo (JCGM 101, JCGM 102) and Bayesian calculations yield a PDF for the measurand

- Report
  - Summary statistics,
  - PDF, or
  - Both





- Summary statistics for univariate measurand
  - Expectation (mean) as estimate y
  - Standard deviation as standard uncertainty $u(y)$
  - Coverage interval extracted from PDF
  - No natural concept of ‘expanded uncertainty’
  - Summary statistics may suffice to report current measurement
  - Possibly inadequate in a further stage of Monte Carlo or Bayes

PDF as a statement of a measurement result

EA/4-02 example S9: calibration of hand-held digital multimeter (DMM)

Additive measurement model $E_{\mathrm{X}} = V_{\mathrm{iX}} - V_{\mathrm{S}} + \delta V_{\mathrm{iX}} - \delta V_{\mathrm{S}}$

V_{iX} is voltage indicated by DMM (index i means indication)
V_{S} is voltage generated by the calibrator
δV_{iX} is correction of indicated voltage due to DMM's finite resolution
δV_{S} is correction of the calibrator voltage due to drift, etc.

PDF as a statement of a measurement result

EA/4-02 example S9: calibration of hand-held digital multimeter (DMM)

Additive measurement model $E_\mathrm{X} = V_\mathrm{iX} - V_\mathrm{S} + \delta V_\mathrm{iX} - \delta V_\mathrm{S}$

V_iX is voltage indicated by DMM (index i means indication)
V_S is voltage generated by the calibrator
δV_iX is correction of indicated voltage due to DMM's finite resolution
δV_S is correction of the calibrator voltage due to drift, etc.

quantity X_i	estimate x_i	standard uncertainty $u(x_i)$	probability distribution	sensitivity coefficient c_i	uncertainty contribution $u_i(y)$
V_iX	100,1 V	-	-	-	-
V_S	100,0 V	0,001 V	normal	-1,0	-0,001 V
δV_iX	0,0 V	0,029 V	rectangular	1,0	0,029 V
δV_S	0,0 V	0,0064 V	rectangular	-1,0	-0,0064 V
E_X	0,1 V				0,030 V

Uncertainty budget (ack. EA)

PDF as a statement of a measurement result (continued)

Dominant contribution V_{iX}, having a rectangular distribution

$u(E_{\mathrm{X}}) = 0.030\,\mathrm{V}$ scarcely larger than $u(\delta V_{\mathrm{iX}}) = 0.029\,\mathrm{V}$

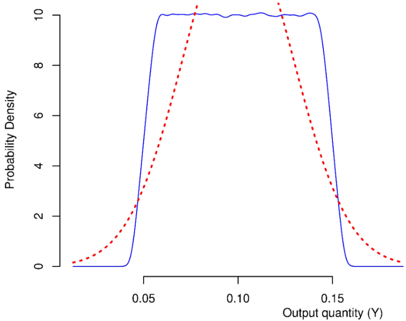
Expect probability distribution for V_{iX} would be close to rectangular

PDF as a statement of a measurement result (continued)

Dominant contribution V_{iX}, having a rectangular distribution

$u(E_{\mathrm{X}}) = 0.030\,\mathrm{V}$ scarcely larger than $u(\delta V_{\mathrm{iX}}) = 0.029\,\mathrm{V}$

Expect probability distribution for V_{iX} would be close to rectangular



PDF (continuous blue line) for E_{X} given by MC and normal PDF (broken red line) with same mean and standard deviation

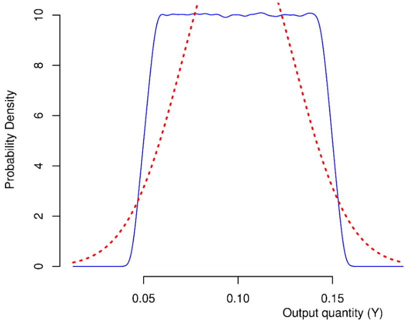
(Ack. NIST Uncertainty Machine: `https://uncertainty.nist.gov/`)

PDF as a statement of a measurement result (continued)

Dominant contribution V_{iX}, having a rectangular distribution

$u(E_{\mathrm{X}}) = 0.030\,\mathrm{V}$ scarcely larger than $u(\delta V_{\mathrm{iX}}) = 0.029\,\mathrm{V}$

Expect probability distribution for V_{iX} would be close to rectangular



PDF (continuous blue line) for E_{X} given by MC and normal PDF (broken red line) with same mean and standard deviation
(Ack. NIST Uncertainty Machine: `https://uncertainty.nist.gov/`)

All information necessary to apply GUM <u>and</u> MC present in uncertainty budget

Numerical accuracy and rounding

- Often claimed that two significant digits sufficient for reporting measurement uncertainty
- Frequently true but there are exceptions

Numerical accuracy and rounding

- Often claimed that two significant digits sufficient for reporting measurement uncertainty
- Frequently true but there are exceptions
- Reporting covariances or correlations
  - When correlation coefficients near ± 1 more digits are required
  - A correlation coefficient of 0.996 should not be reported as 1 since that implies dependence
  - Number of digits that should be quoted depends on eigenvalues of covariance matrix!
  - Recommend holding all uncertainty information to full numerical precision
  - Subsequent calculations can proceed safely
  - Values actually reported can be rounded but should only be used in subsequent calculations with great care

Concluding remarks

- Reporting estimates and standard uncertainties uncontroversial
- Reporting estimates and expanded uncertainties misleading for asymmetric PDFs
- PDF as a statement of a measurement result
- Acceptance in some quarters will be a problem
- Hopefully the coming of e-certificates will facilitate use
- Need to consider carefully ways of ‘selling’ the concept
- Multistage evaluations: take care of rounding

The EMPIR initiative is co-funded by the European Union's Horizon 2020 research and innovation programme and the EMPIR Participating States

Examples of Measurement Uncertainty Evaluation

This project has received funding from the EMPIR programme co-financed by the Participating States and from the European Union's Horizon 2020 research and innovation programme