Objections against uncorrected systematic effects

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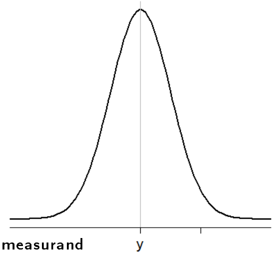
Best practice

- measurement to quantify measurand Y
- assign value (estimate) and quality measure (uncertainty)

GUM

- $Y = f(X_1, \ldots, X_n)$, characterize inputs, propagate uncertainties, ...
- best estimate y



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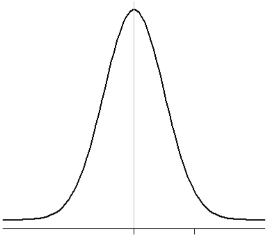

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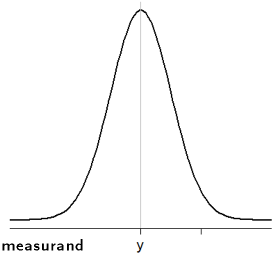
    Note 2 in 3.2.4, 6.3.1, F.2.4.5



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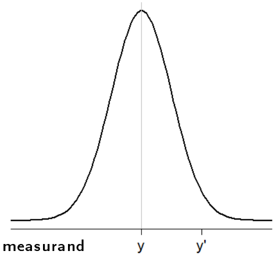

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  3. calculate uncertainty $u(y') = \sqrt{u^2(y) + (y - y')^2}$ [Lira and Wöger, 1998]
  4. various suggestions for expanded uncertainty DKD-R 3-3, 3-5, 3-6, 3-7, 3-8, 3-9, 6-1, [EURAMET, 2011, EURAMET, 2015, ISO/TC 158, 2005, Phillips et al., 1997, Haesselbarth, 2004, Synek, 2005, Magnusson and Ellison, 2008, Hernla, 2008]

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$\Rightarrow$ reasonable, or not?

Theory

Properties: best/other estimates

best estimate y statist. def. e.g. [Robert, 2007]

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? $(y', u(y'))$ reasonable metrologically?

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- law of propagation of uncertainty

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$$= u^2(y) + \left(y - y'\right)^2 + 2\left(y_1 - y_1'\right)\left(y_2 - y_2'\right) \neq u^2(y')$$

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  as $a^2 + c^2 \neq (a+c)^2$
- law of propagation of uncertainty: requires also y_i'

Conclusions

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Publication Klauenberg, Wübbeler and Elster. (2019). About not correcting for systematic effects. Measurement Science Review, 19(5):204–8.

Recommendation

- revision of relevant standards & guidelines

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References

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