BAYESIAN UNCERTAINTY ANALYSIS OF INVERSION PROBLEM APPLIED TO THERMAL MEASUREMENTS

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OUTLINE

Motivation

Presentation of the case study

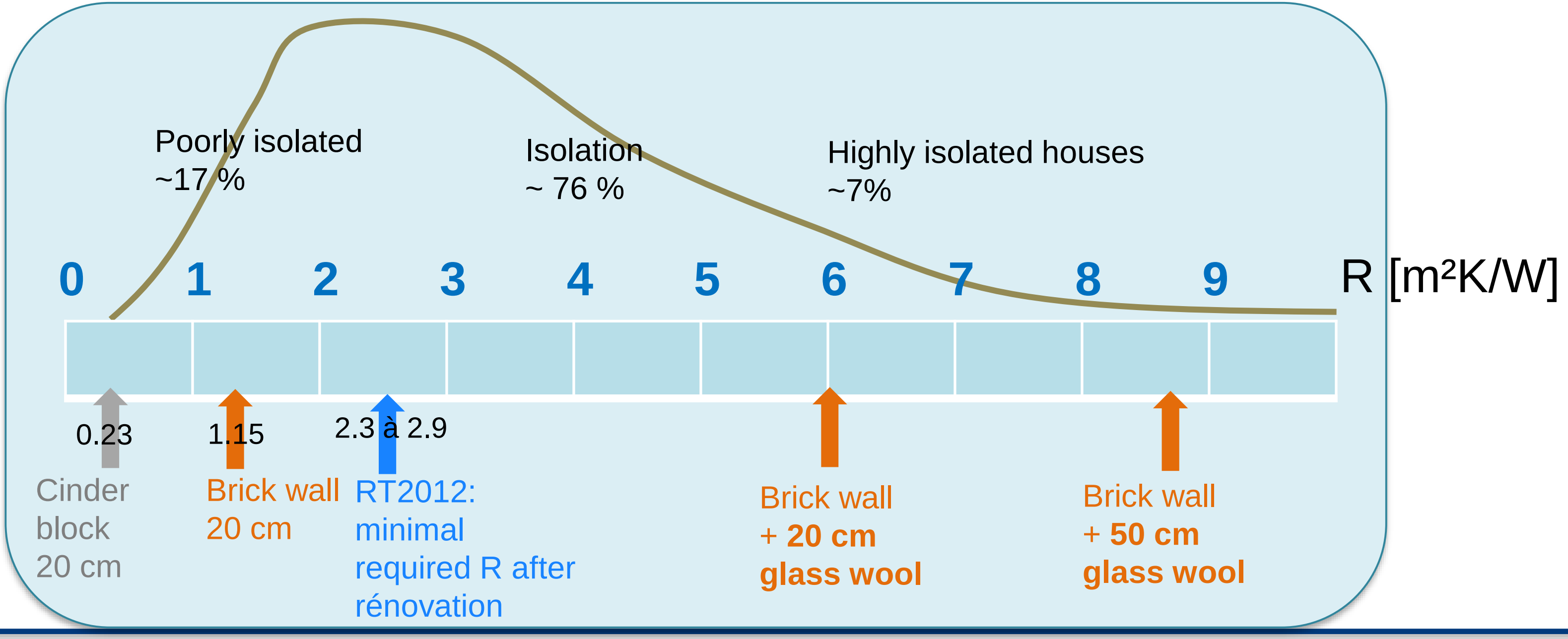
Bayesian inversion model

Results and conclusion

MOTIVATION

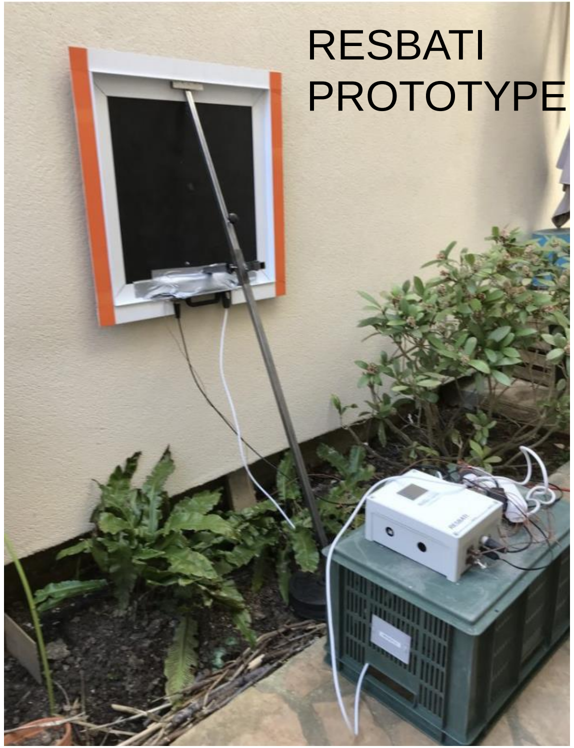
In France, the Building sector represents 40% of the energy consumption and 25% of carbon dioxide emissions → renovation through isolation

Typical values of **thermal resistance** and repartition of french housing in 2018



OBJECTIVE OF THE FRENCH ANR RESBATI CONSORTIUM (2017-2021)

To develop a prototype for the **in-situ evaluation of the thermal performance** of opaque walls and its **associated uncertainty**.



REQUIREMENTS

- Portable
- Non intrusive
- Fast (<1day)
- Level of uncertainty comparable or lower than with a reference equipement (guarded hot box or guarded hot plate)

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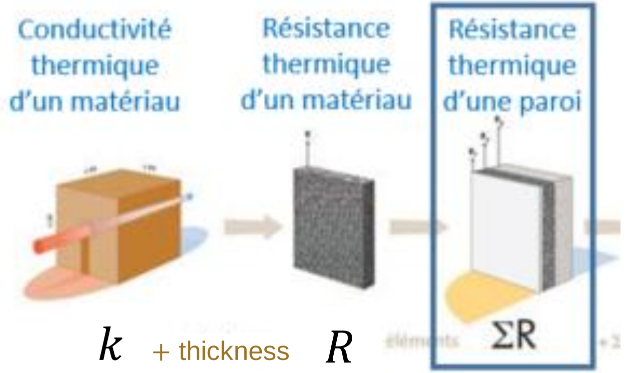
QUANTITY OF INTEREST: THERMAL RESISTANCE OF THE WALL R

$$\boldsymbol{R = l/k}$$

$$R = \sum_{i=1}^{I} R_i, \text{with } R_i = \frac{l_i}{k_i}$$



i : index of layer of the wall (here $i = 1, \ldots, 4$)
R_i: thermal resistance of the layer
k_i: thermal conductivity of the layer
l_i: thickness of the layer

<u>Objective</u> : To estimate the posterior distribution of the k_i.

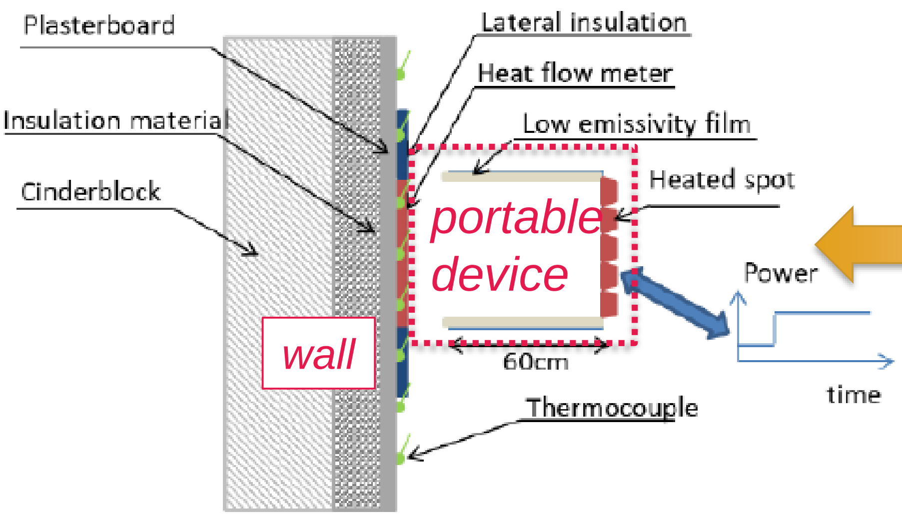
The **posterior distribution of R** is obtained with Monte Carlo sampling in the distribution of the k_i

CASE STUDY

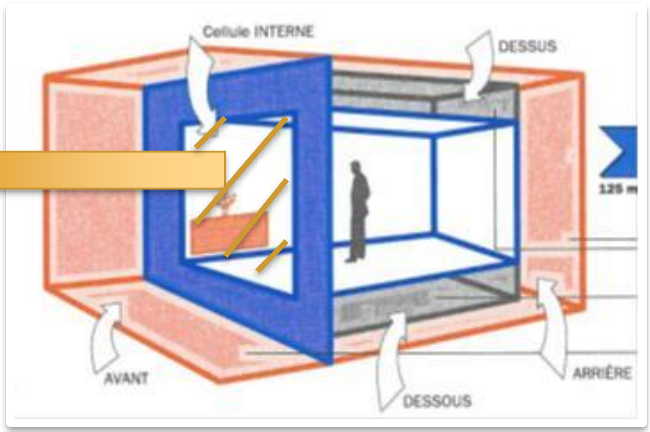
SCHEMATIC VIEW OF THE EXPERIMENTAL SETUP IN THE LNE ENERGY ROOM REBECCA

Total thickness : 30 cm
EPS : 12 cm

side view



ENERGY ROOM



→ indirect measurement of the thermal resistance from surface temperatures and absorbed flux measurements → **inversion problem under uncertainty**

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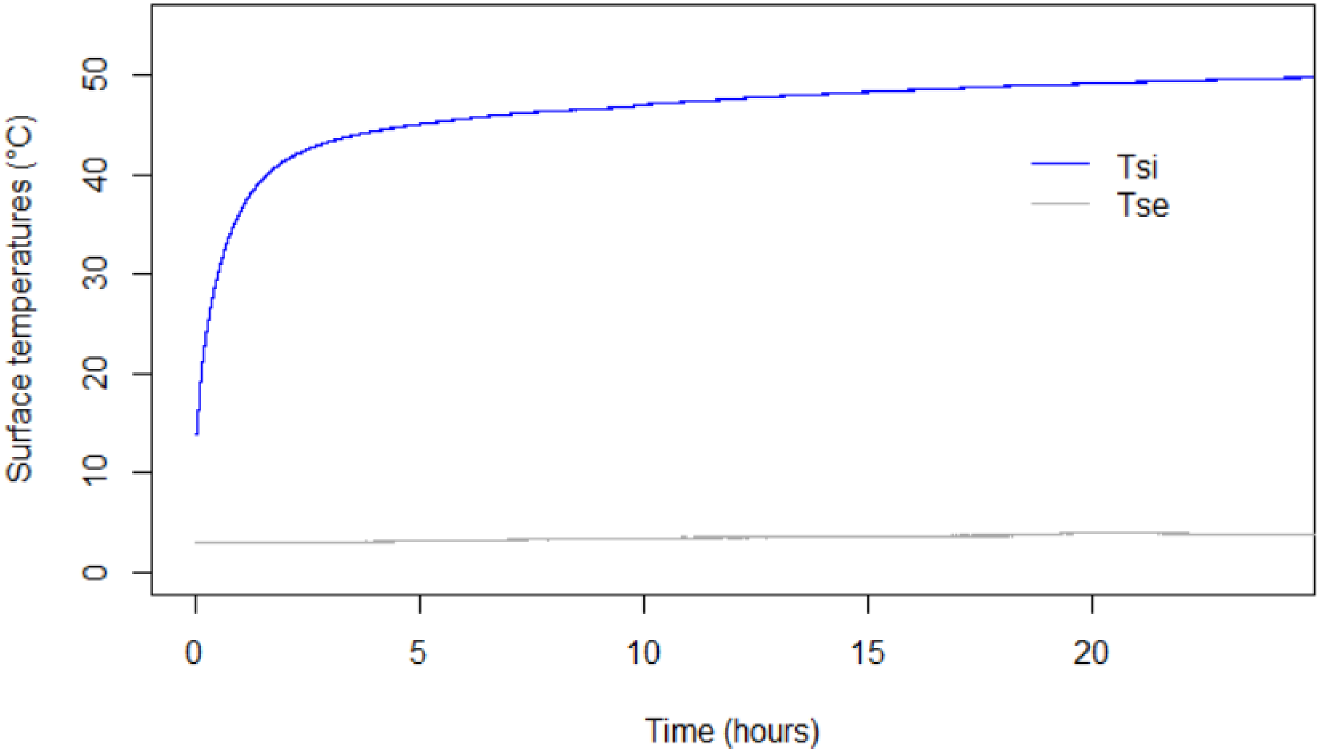
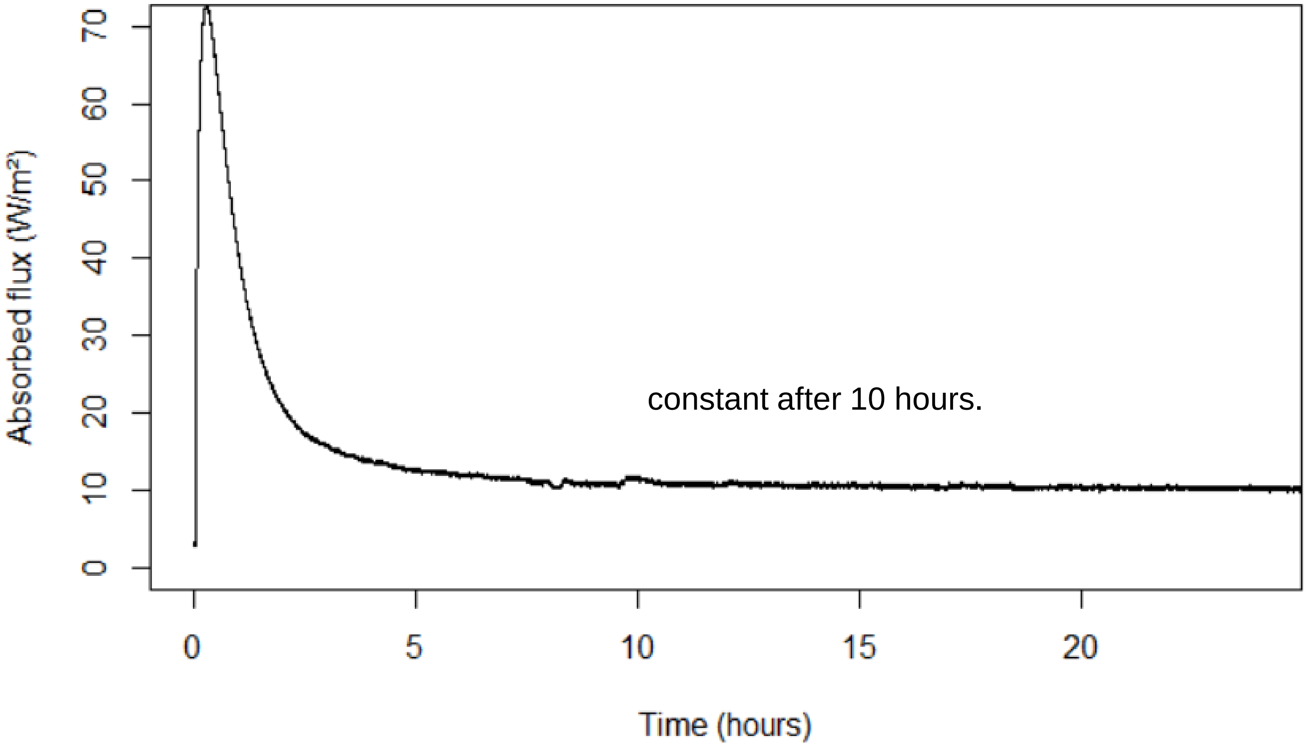
SURFACE TEMPERATURE MEASUREMENTS



ABSORBED FLUX MEASUREMENTS



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EXPERIMENTAL SETTINGS USED FOR VALIDATION

Guarded hot plate at LNE for layer 2 : EPS (expanded polystyrene)

Layer	Heat capacity of unit area cw /$\mathrm{J\,m^{-3}\,K}$	Thermal conductivity k /$\mathrm{W\,m^{-1}\,K^{-1}}$	Thermal resistance R /$\mathrm{m^2\,K\,W^{-1}}$
EPS	$cw_2 = 1.35 \times 10^4$	$k_2 = 0.031$	$R_2 = 3.85$
EPS + Plasterboard	-	-	3.9

1% expanded uncertainty!

Guarded hot box for the global resistance

95% confidence interval: $\hat{R} \approx 4.08 \pm 0.86\, m^2 K W^{-1}$

$\boxed{\text{Remark: } R_2 \approx 95\% R}$

20% expanded uncertainty!

BAYESIAN INVERSION MODEL

INVERSION PROBLEM TO GET THE POSTERIOR DISTRIBUTION OF $\{k_i\}$

θ, calibration parameters
(conductivity, heat capacity,...)
Quantity of interest k_i

x, input parameters
(boundary and initial conditions,...)
$Tse,\ q_{int}$

Reported measurement uncertainty

thermal model
simulator η

$\eta(x, \theta)$, scalar output
(temperature)

$y(x)$, scalar observation
(temperature)
Tsi

Reported measurement uncertainty

Model: $y = \eta(X, \theta) + \sigma\varepsilon$ $\qquad\qquad$ $\varepsilon_i \sim N(0, u_i)$

Excess variance parameter
to account for measurement noise

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SUMMARY OF BAYESIAN UNCERTAINTY FRAMEWORK FOR INVERSION

Take advantage of the Bayesian paradigm by assigning prior distributions to all uncertain quantities in addition to calibration parameters.

Set an computational algorithm to sample from the joint posterior distribution $\pi(X, \theta, \sigma | y)$

Ckeck for convergence of the posterior samples (autocorrelation, tests of convergence,…) towards the stationary distribution.

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PRIOR KNOWLEDGE FOR CALIBRATION PARAMETERS θ

Boundaries on calibration parameters are usually used to facilitate the search.

Layer	Thickness /m	Thermal conductivity k /$\mathrm{W\,m^{-1}\,K^{-1}}$	Heat capacity of unit area cw /$\mathrm{J\,m^{-2}\,K^{-1}}$
Plasterboard	$l_1 = 0.013$	$0.2 \leq k_1 \leq 0.4$	$7 \times 10^5 \leq cw_1 \leq 8 \times 10^5$
EPS	$l_2 = 0.120$	$0.02 \leq k_2 \leq 0.04$	$1 \times 10^4 \leq cw_2 \leq 3 \times 10^4$
Cinderblock	$l_3 = 0.150$	$0.7 \leq k_3 \leq 1.2$	$8.5 \times 10^5 \leq cw_3 \leq 2 \times 10^6$
Exterior coating	$l_4 = 0.015$	$0.5 \leq k_4 \leq 1.2$	$1 \times 10^6 \leq cw_4 \leq 2 \times 10^6$

Here, prior distributions of calibration parameters are chosen rectangular.

PRIOR KNOWLEDGE FOR INPUT PARAMETERS X

Here, measurement uncertainty is obtained from GUM uncertainty propagation.

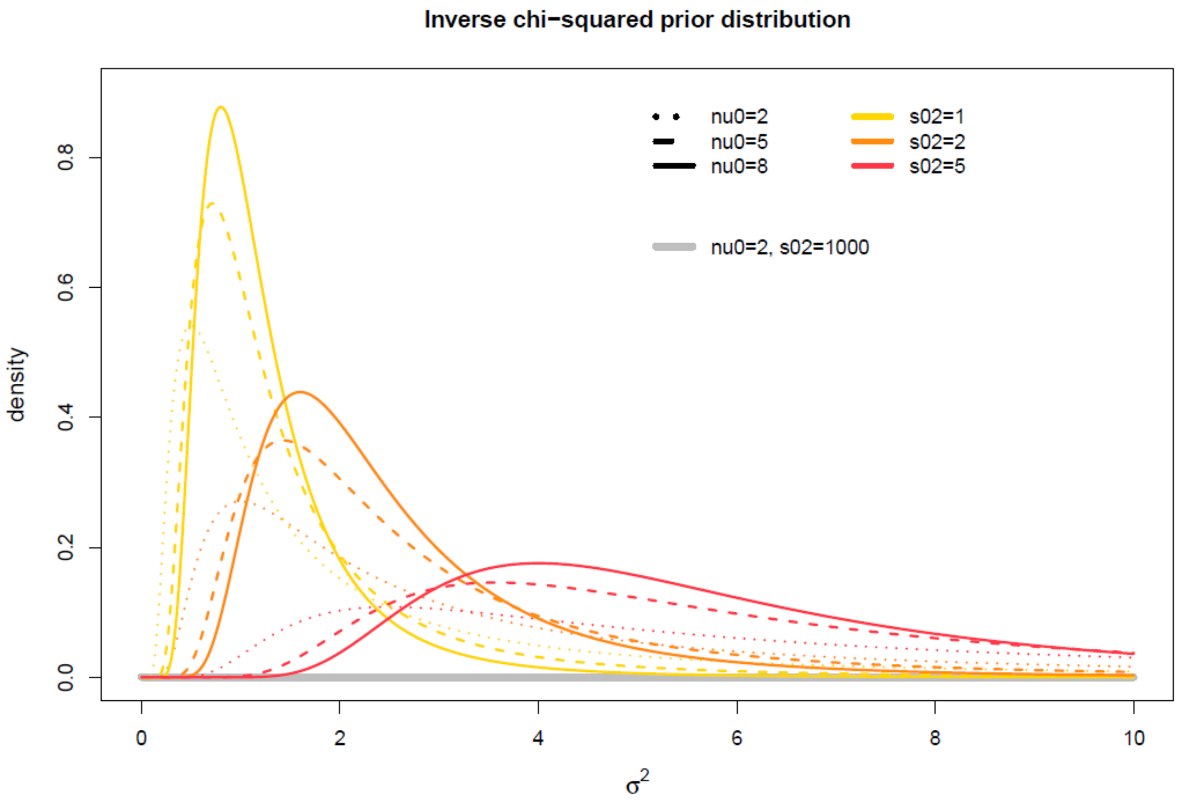
Tse, q_{int} are Gaussian random variables whose uncertainty expresses deviation from measurements.
The following error-in-variables representation is used

$$Tse = tse_{observed} + error_{Tse}$$
$$q_{int} = q_{observed} + error_{q_{int}}$$

with $error_{Tse} \sim N\left(0, u_{Tse}\right)$ and $error_{q_{int}} \sim N(0, u_{q_{int}})$

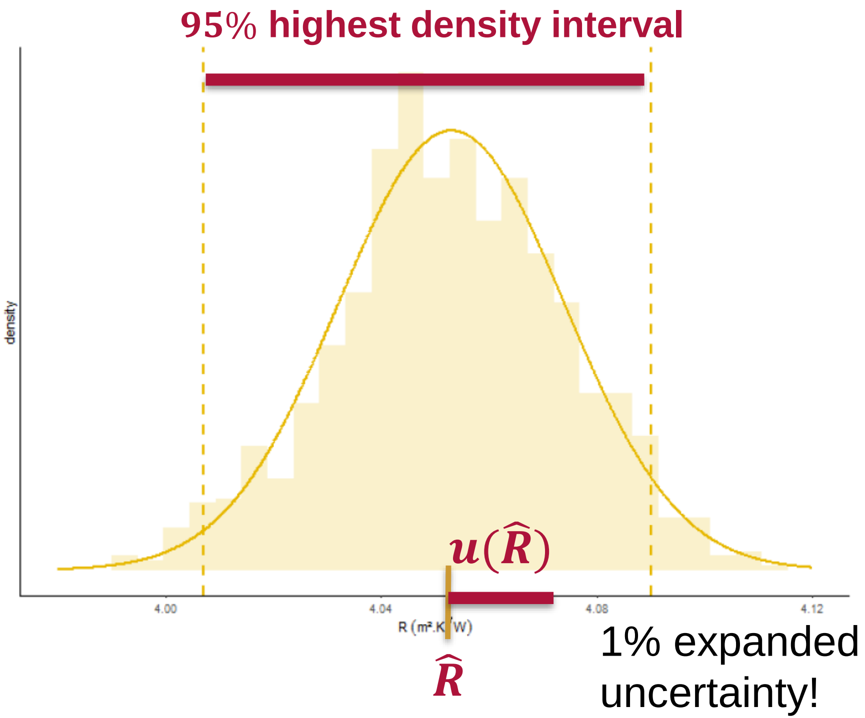
PRIOR DISTRIBUTION OF σ

$$\sigma^2 \sim Inverse - Chi2(\nu_0, s_0^2)$$



RESULTS

BAYESIAN APPROACH GIVES A POSTERIOR DISTRIBUTION OF R GIVEN ALL AVAILABLE INFORMATION



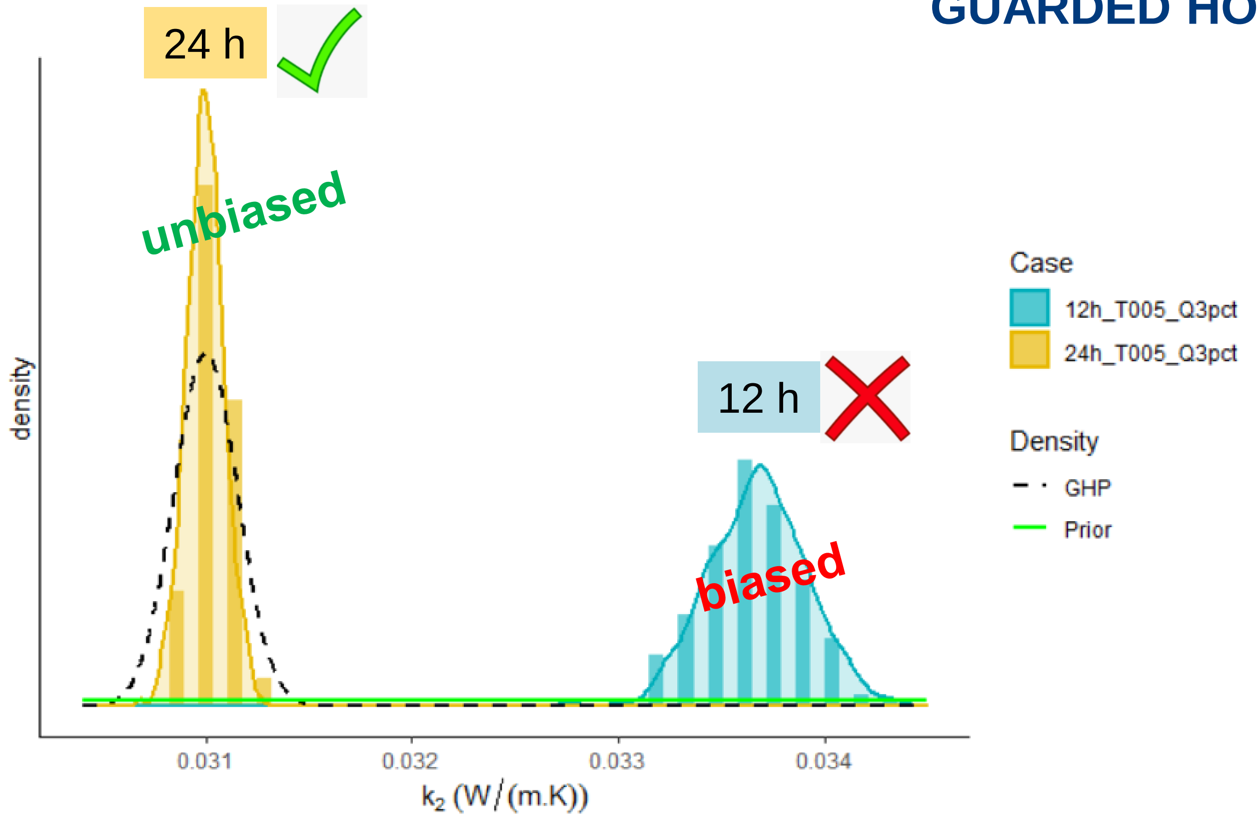
The posterior distribution combines all uncertainty sources according to prior and expert knowledge and statistical modelling.

→ Possibility to retrieve point estimates $\hat{R}$ and $u(\hat{R})$, coverage intervals

→ Allows non Gaussian QoI (here slight deviation from Gaussian in the tails)

BUT high computational price

EFFECT OF DURATION ON k_2 : 12 H VERSUS 24 H (COMPARISON WITH GUARDED HOT PLATE)



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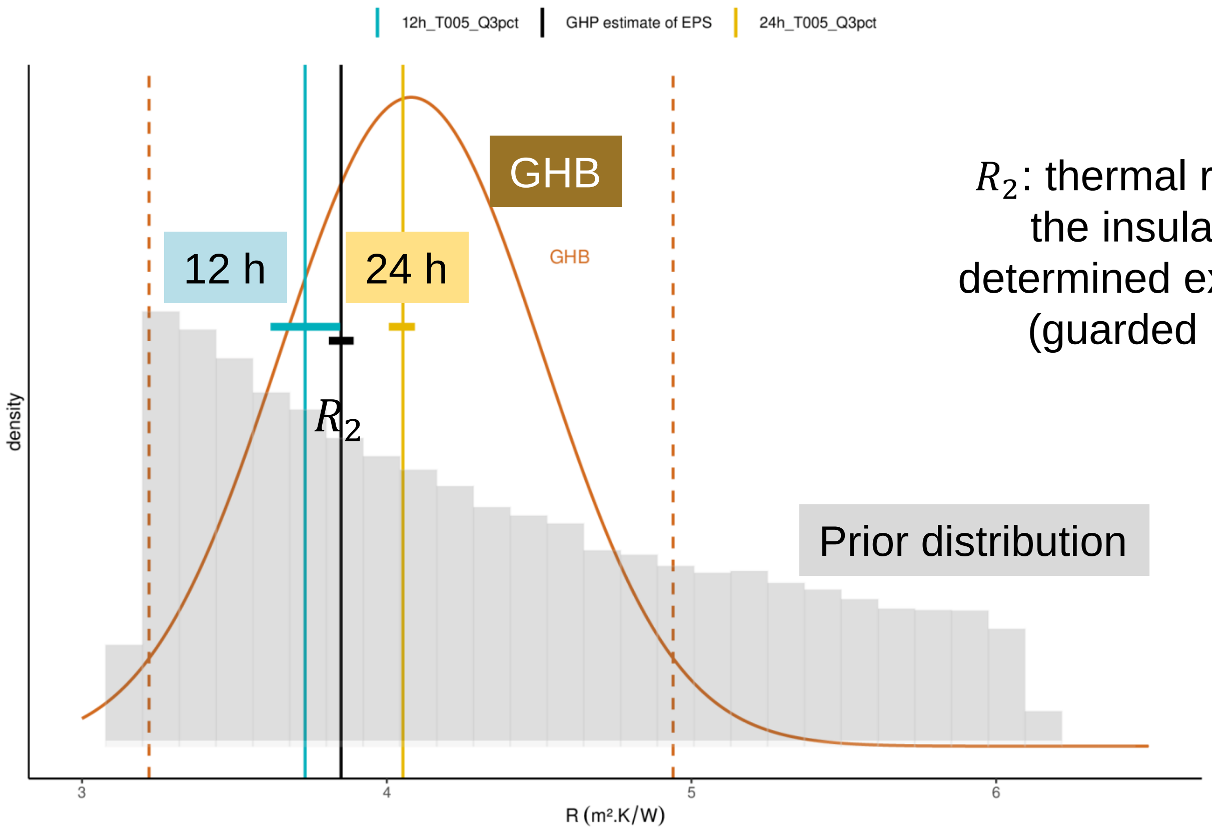
SUMMARY OF RESULTS FOR R (COMPARISON WITH GUARDED HOT BOX)



R_2: thermal resistance of the insulation layer determined experimentally (guarded hot plate)

CONCLUSION AND PERSPECTIVES

Bayesian approach is used here to ***propagate input uncertainties in an inversion problem***, which is in practice an issue rarely addressed, with no consensus methodology, so far up to our knowledge.

Proposed solution : to take all uncertainty inputs as calibration parameters and to update their posterior distribution.

The MCMC method is computationally expensive when used in combination with a numerical code. In practice, strong autocorrelation of Markov chains may appear which makes the interpretation of results less reliable (mostly for the posterior uncertainty).

Perspectives concern taking into account lateral fluxes when heating the wall with an improved thermal model. Due to the prohibitive expected cost of the estimation method, surrogates of the thermal model will be introduced.

Thanks for your attention