

GUM-compliant propagation of conformity statements and maximum permissible errors

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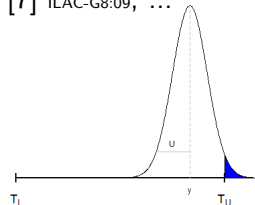
²WG Communications Technology of Weighing Instruments, Physikalisch-Techn. Bundesanstalt

Conformity assessment

- ▶ decide whether an item conforms to requirements (tolerance intervals)
- ▶ according to [9] ISO 17025, [13] OIML G 19, [17] WELMEC 4.2, [7] ILAC-G8:09, ...

Conformity statements

- ▶ risk-based, account for uncertainty [9]
- ▶ minimum information [9]
 - which specifications are (not) met
 - the decision rule applied



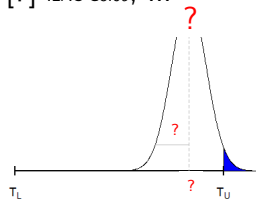
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⇒ often unavailable: measured value, uncertainty (or distribution)



Conformity assessment

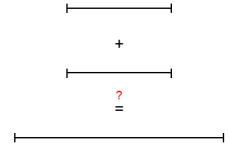
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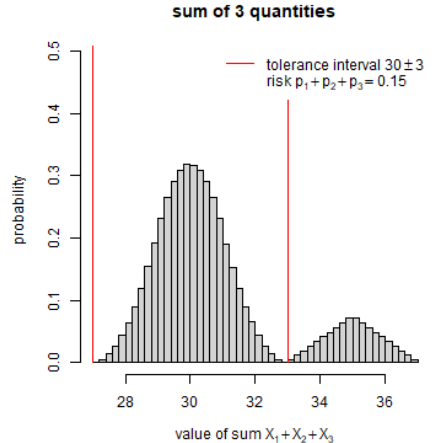
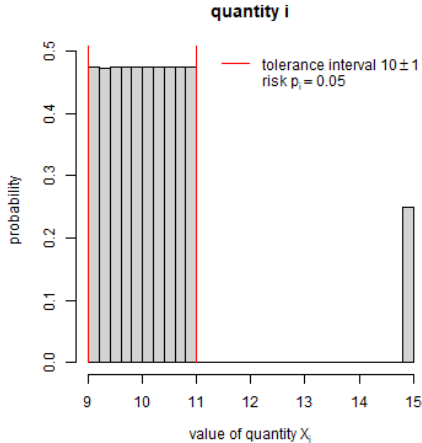
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 - which specifications are (not) met
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Question: Can we reuse conformity statements to disseminate traceability?





\Rightarrow risks of non-conformance may accumulate

Introduction

Introduction to Conformity Statements

Insufficient Knowledge

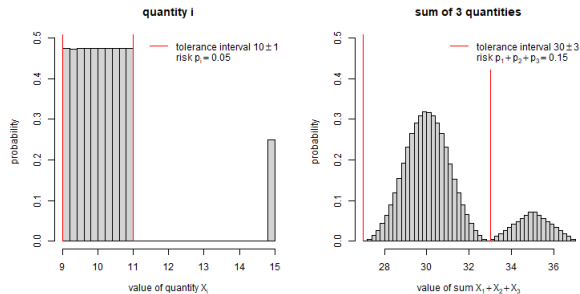
Need and Content

Review of Guidance to Propagate Conformity Statements

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Lemma

Let $P(Y_i \notin y_i \pm MPE_i) = p_i$ and $c_i \neq 0$ for $i = 1, 2, \dots$. Then the risk $P(\sum c_i Y_i \notin \sum c_i y_i \pm \sum |c_i| MPE_i) \leq \min(\sum p_i, 1)$ and equality may hold.

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Introduction to Conformity Statements

Insufficient Knowledge

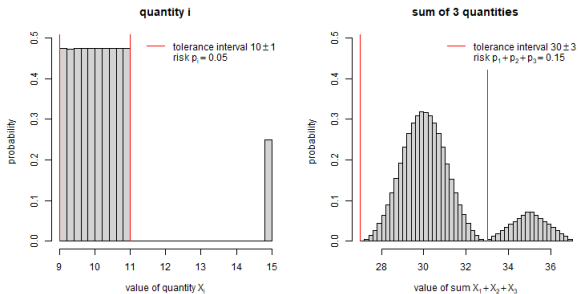
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- knowledge only on conformity for each input is insufficient to judge conformity of a combination of these inputs
also not compliant with [9], but applied in practice [15, 4]

Conformity statements needed

- ▶ for the linear combination of quantities for which in turn conformity statements are available
- ▶ which comply with [9] and [3] JCGM 106: account for risk and uncertainty
- ▶ in legal metrology: to correct [15]
e.g. long vehicle weighing, tara weighing in zero waste shops, ...
source: de.m.wikipedia.org/wiki/Datei:LKW_mit_Aufleger_aus_Zusatzzeichen_1048-14.svg



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Content

- ▶ brief review of available guidance [1, 2, 3]
- ▶ develop simple, risk-based decision rules
based on specifications and decision rule or risk for each input
- ▶ illustrate by example

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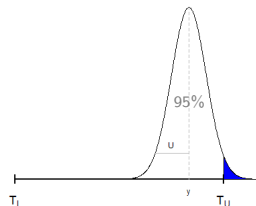
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Conformity guidance in [3]: measurand expressed by

- ▶ probability density function (pdf), or
- ▶ estimate, coverage interval and probability

Guidance in [2] JCGM 101 on output pdf based on



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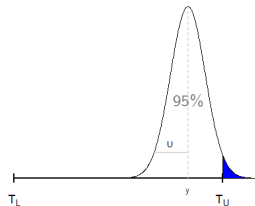
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- ▶ pdf for inputs
- ▶ MC method

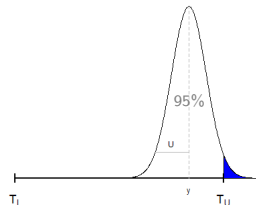


Conformity guidance in [3]: measurand expressed by

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- ▶ pdf for inputs \Rightarrow case by case evaluation
- ▶ MC method \Rightarrow not widespread at end of traceability chain [14]

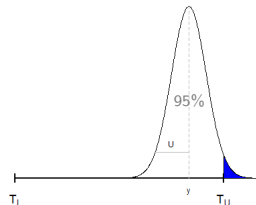


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Guidance in [1] JCGM 100 on output pdf based on

- ▶ estimate, uncertainty and degrees of freedom for inputs
- ▶ Normal or t -distribution for output:
verify CLT assumptions or Normality and independence of inputs
- ▶ apply LPU, calculate coverage factor



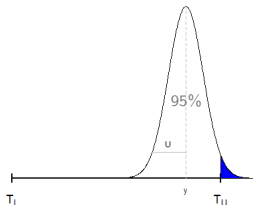
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⇒ available for conformity statements based on calibr. certificates [8, 5]



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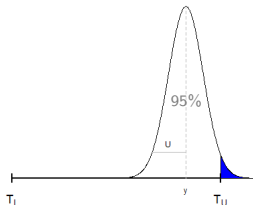
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⇒ available for conformity statements based on calibr. certificates [8, 5]

⇒ methods needed for conformity statements with less info



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Theorem (Sum of Normal and Uniform Distributions)

Let $U_i \sim U(-MPE_i, MPE_i)$ and $X_i \sim N(0, (MPE_i/m)^2)$ be independent with $i = 1, \dots, n$, $m > 1$. Then calculation of the quantile $-\sum MPE_i$ of their sum $\sum_{i=1}^n (U_i + X_i)$ reduces to an essentially 1-dim. numerical integration

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$$P(\bar{U} + \bar{X} < -MPE) = \int_{-MPE}^{MPE} f_{\bar{U}}(v) \Phi\left(\frac{-m(v + MPE)}{\sqrt{\sum MPE_i^2}}\right) dv .$$

with $MPE = \sum MPE_i$ and Φ the standard Normal CDF

- as $\bar{U} = \sum_{i=1}^n U_i$ and $\bar{X} = \sum_{i=1}^m X_i$ analytic
for non-id. distributed U_i : generalization of Irwin-Hall distribution [16]

Decision rule I assumes:

- ▶ measurements for each input Y_i conform to specification $\pm \text{MPE}_i$
- ▶ conformity was established by an instrument with $X_i \sim N(0, u_i^2)$ and $u_i \leq \text{MPE}_i/m$ (for $m > 1$)
- ▶ $Y_i = X_i + U_i$ for each $i = 1, \dots, n$
- ▶ independence of all quantities involved

Decision rule I assumes:

► measurements for each input Y_i conform to specification $\pm \text{MPE}_i$
 $\Rightarrow U_i \sim u^{(i)} + U(-\text{MPE}_i, \text{MPE}_i)$ cf. [1, 2]

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\Rightarrow apply Theorem to calculate risk of $\sum c_i Y_i$ not conforming to $\pm \sum |c_i| \text{MPE}_i$

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\Rightarrow apply Theorem to calculate risk of $\sum c_i Y_i$ not conforming to $\pm \sum |c_i| \text{MPE}_i$

\Rightarrow complies with the GUM [1, 2, 3] and ISO 17025 [9]

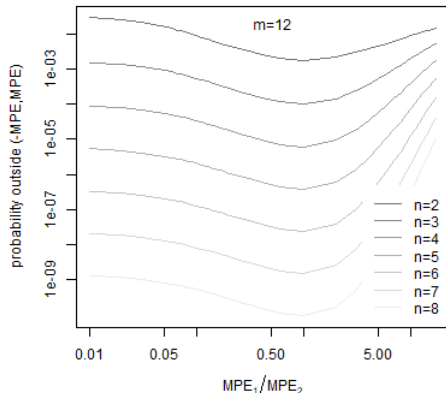
\Rightarrow applicable also for small n and bounds of u_i , not so guidance in [1]

\Rightarrow risks smaller than for distribution-free coverage intervals

Decision rule I simplified:

tabulate risk for common settings

- ▶ $\text{MPE}_1 = \dots = \text{MPE}_n, c_i = \pm 1$:
depends only on n (and m)
- ▶ $\text{MPE}_2 = \dots = \text{MPE}_n, c_i = \pm 1$:
depends only on $n, \frac{\text{MPE}_1}{\text{MPE}_2}$ (and m)
- ▶ for small n
- ▶ ...



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with $X_i \sim N(0, u_i^2)$ and $u_i \leq \text{MPE}_i/m$ (for $m > 1$)
 \Rightarrow available from calibration certificate for X_i with $k = 2$, or
 \Rightarrow derive from decision rule for Y_i [6, 13]:
 - simple acceptance: often U limited with $k = 2$, e.g. $U \leq \text{MPE}/3$ [3]
 - guarded acceptance: e.g. $U \leq A_L - T_L$



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□ $Y_i = X_i + U_i$ for each $i = 1, \dots, n$

□ independence of all quantities involved



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Decision rule II assumes:

- ▶ measurements for each input Y_i conform to specification $\pm \text{MPE}_i$
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Lemma

Let $P(Y_i \notin y_i \pm \text{MPE}_i) = p_i$ and $c_i \neq 0$ for $i = 1, 2, \dots$. Then the risk $P(\sum c_i Y_i \notin \sum c_i y_i \pm \sum |c_i| \text{MPE}_i) \leq \min(\sum p_i, 1)$ and equality may hold.

Decision rule II assumes:

- ▶ measurements for each input Y_i conform to specification $\pm \text{MPE}_i$
- ▶ risk of non-conformance $\leq p_i$ for each input

$\Rightarrow \sum c_i Y_i$ conforms to $\pm \sum |c_i| \text{MPE}_i$ if $\sum p_i$ small (see Lemma)

\Rightarrow very simple, correlation may be unknown

\Rightarrow complies with JCGM 106 [3] and ISO 17025 [9]

\Rightarrow JCGM 100, 101 [1, 2] and distribution-free intervals not applicable

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Let $P(Y_i \notin y_i \pm \text{MPE}_i) = p_i$ and $c_i \neq 0$ for $i = 1, 2, \dots$. Then the risk $P(\sum c_i Y_i \notin \sum c_i y_i \pm \sum |c_i| \text{MPE}_i) \leq \min(\sum p_i, 1)$ and equality may hold.

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Long vehicle weighing

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Traffic surveillance: overloaded vehicles prohibited

Example of axle load weighing instruments of class IIII [11] with $2.5\text{t} < \text{mass} \leq 10\text{t}$

- ▶ conform typically with $\text{MPE} = 100\text{kg}$, $U \leq \frac{\text{MPE}}{6}$, often $k=2$ [12]

Scenario 1

- ▶ 4 instruments measure simultaneously (independence)
- ▶ decision rule I: risk of $\leq 6 \cdot 10^{-6}$ that the truck weigh measurement doesn't conform to $\pm 400\text{kg}$



Traffic surveillance: overloaded vehicles prohibited

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- ▶ 4 instruments measure simultaneously (independence)
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Scenario 2

- ▶ one instrument measures 4 times (unknown correl.)
- ▶ decision rule II with $p_i \leq 0.033$: there is a risk of ≤ 0.133 that the truck weigh meas. doesn't conform to $\pm 400\text{kg}$
- ☒ currently considered to conform under [15, 4] without limitations



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Conformity statements for linear combinations of quantities

- ▶ showed that knowledge only on conformity for each input is insufficient
- ▶ reviewed GUM guidance \Rightarrow mostly applicable for info in calibr. certificates
- ▶ derived two risk-based decision rules
 - + tailored to little info in conformity statements of inputs (specifications and non-conformance risk or decision rule)
 - + simple to apply for common scenarios
 - + comply with JCGM 106 and ISO 17025
 - + apply to cases where the GUM [1, 2] is ill-suited or not applicable
- ▶ demonstrated need for new, risk-based decision rules in practice

K. Klauenberg, J. Greenwood, and G. Foyer. Propagation of conformity statements in compliance with the GUM and ISO 17025. 2023. Draft available soon

Extension of decision rule I

- ▶ to fixed risks or other quantiles
- ▶ tabulate for other common, simple scenarios
- ▶ develop a WebApp for arbitrary MPE_i
- ▶ to inputs $Y_i = U_i^{(1)} + U_i^{(2)} + X_i$ whose conformity was derived by an instrument whose conformity was in turn established with limited uncert.

Discussion welcome on more needs for decision rules / conformity assessment

Acknowledgment: The authors would like to thank John Greenwood (UKAS) for helpful comments and discussions.

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