Pixel-wise uncertainty quantification in electric properties tomography

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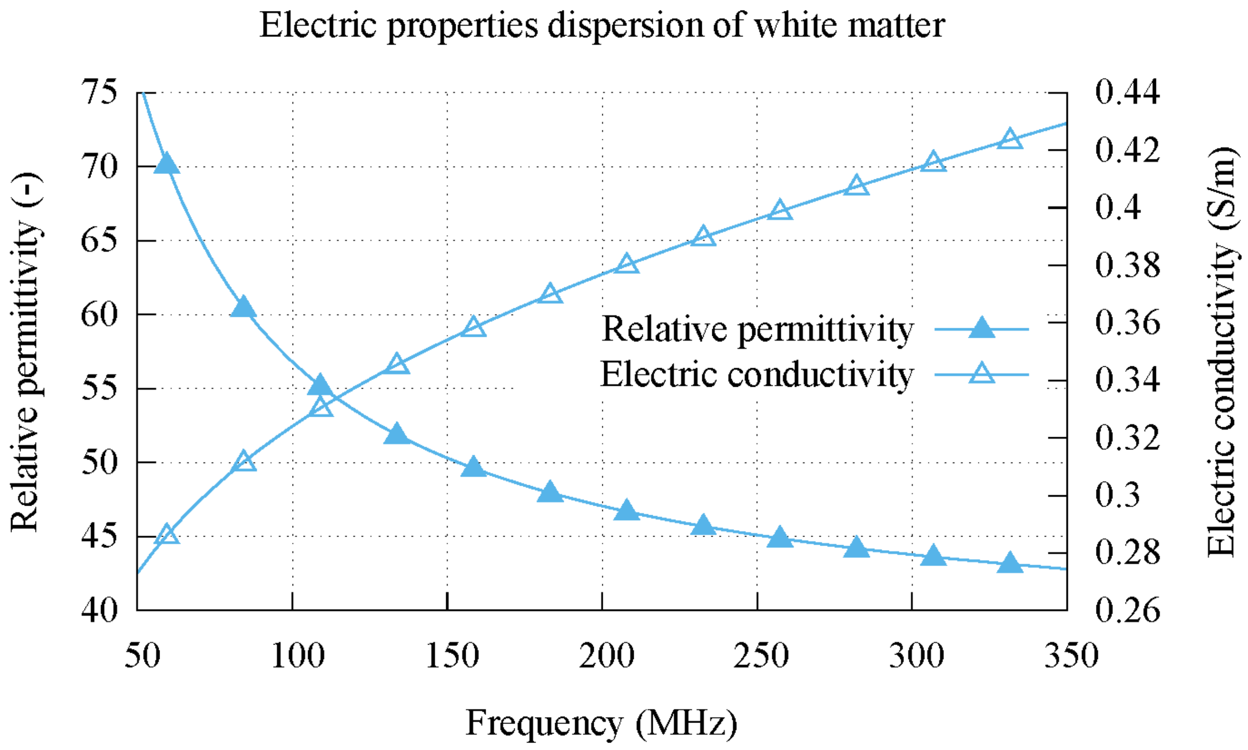
Introduction

- Electric Properties Tomography (EPT) images the electric properties (EPs) of biological tissues

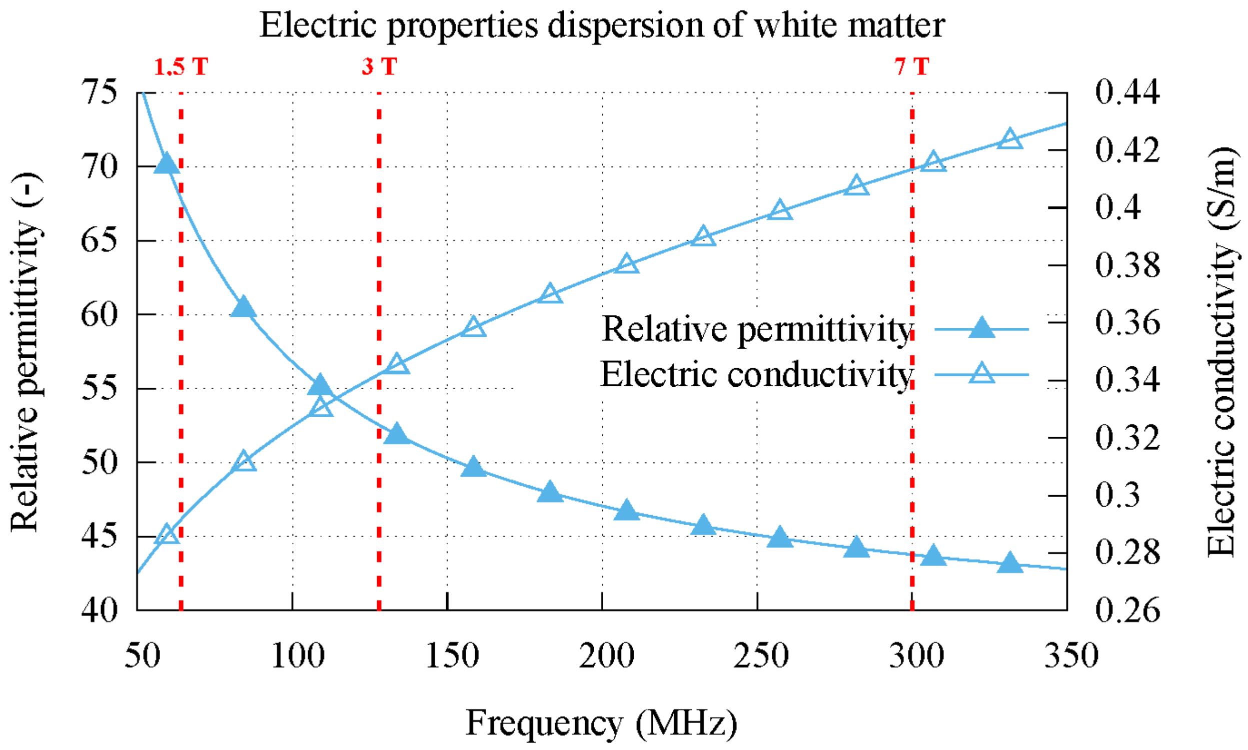


Introduction

- Electric Properties Tomography (EPT) images the electric properties (EPs) of biological tissues

- EPT is based on Magnetic Resonance Imaging (MRI) for the input maps

Static field	Larmor frequency
1.5 T	64 MHz
3 T	128 MHz
7 T	300 MHz



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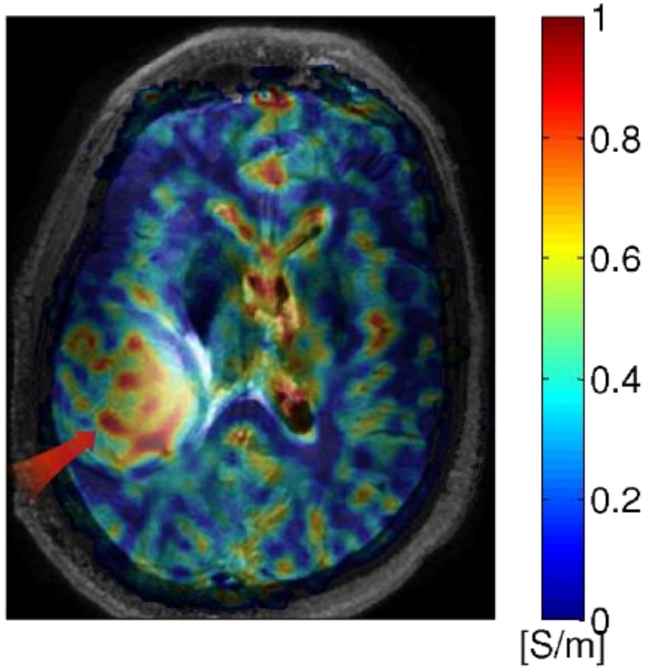
- EPs in the radiofrequency (RF) range can act as physical biomarkers



Van Lier et al., Proc. ISMRM p.4464, 2011

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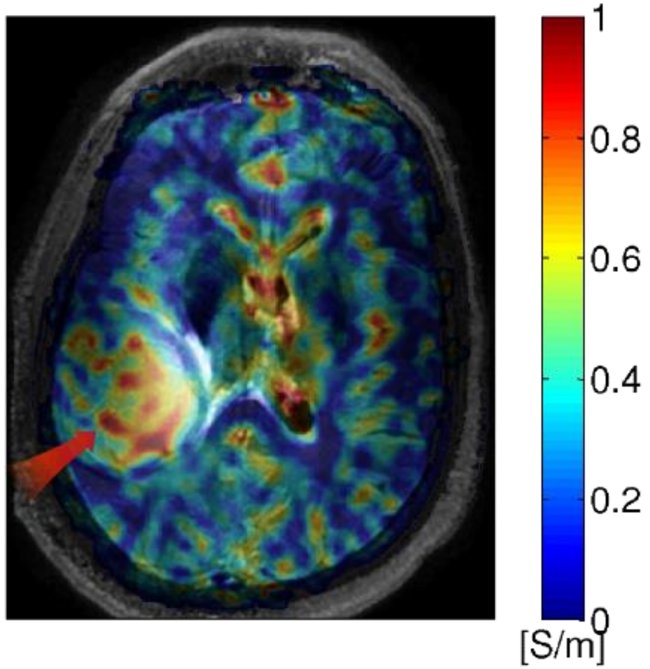
Introduction

- EPs in the radiofrequency (RF) range can act as physical biomarkers
- Knowing the value of tissue EPs is very important for accurate numerical simulations



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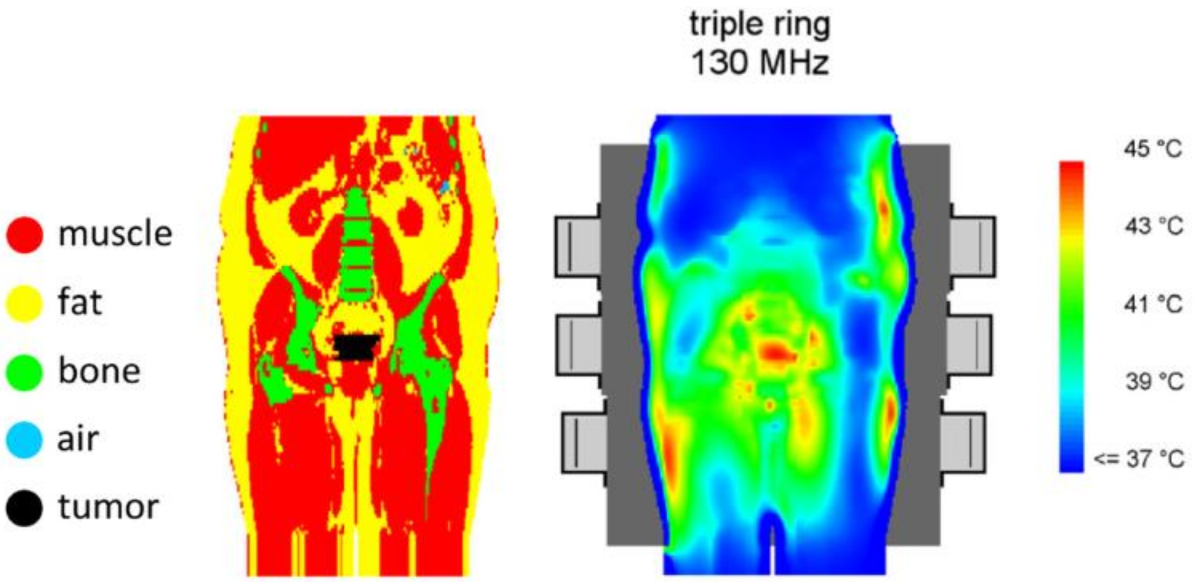

Kok et al., Radiation Oncology 10:196, 2015

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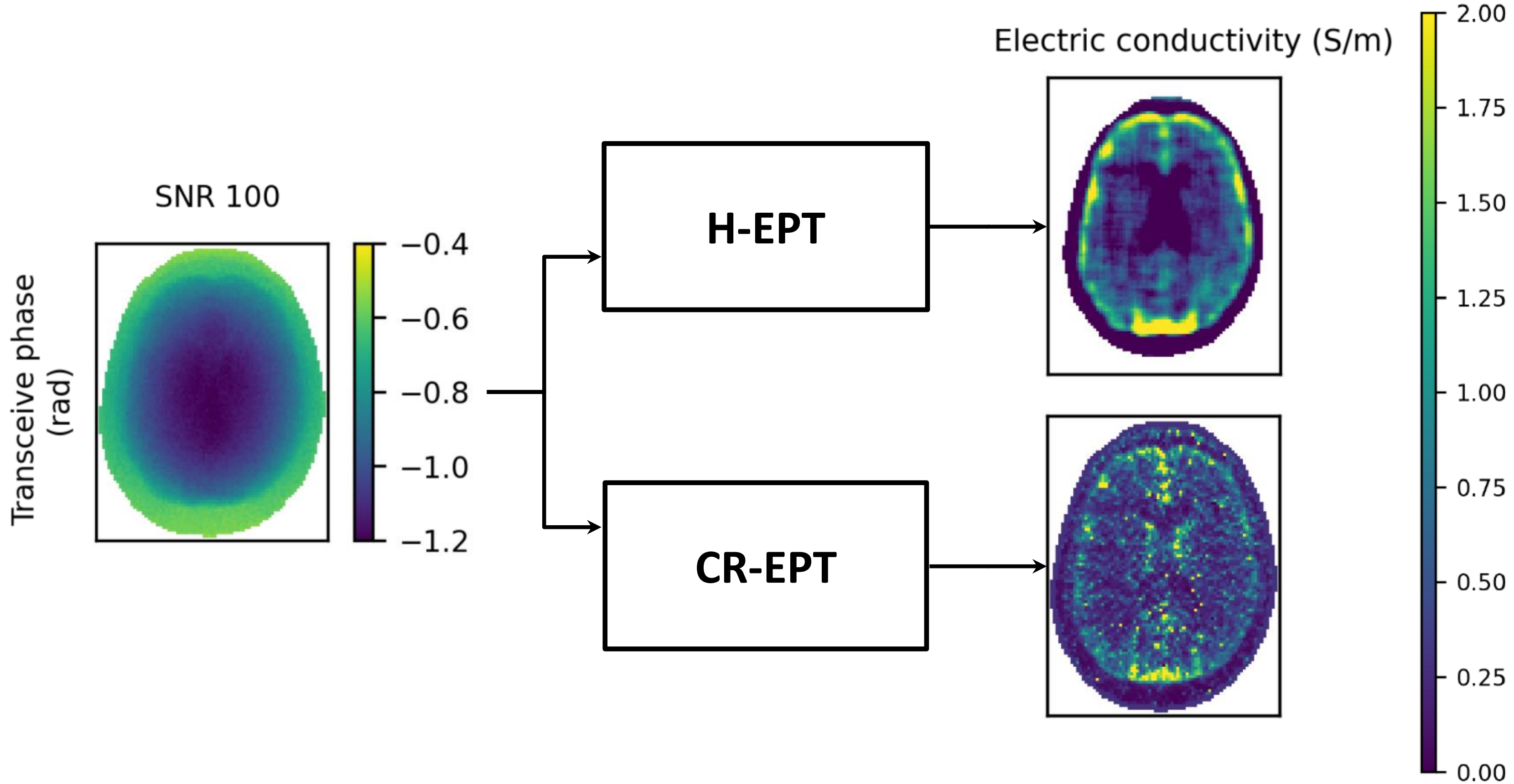
Introduction

- A plethora of techniques for EPT are proposed in the literature

- Each one with its own model errors



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OBJECTIVE

To estimate the **pixel-wise uncertainty** of the retrieved EPs maps taking into account the model errors.

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OBJECTIVE

To estimate the **pixel-wise uncertainty** of the retrieved EPs maps taking into account the model errors.

- This would allow us to compare the EPT results

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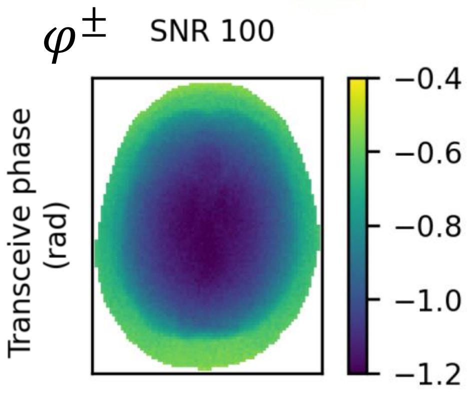
Method

- Phase-based Helmholtz-EPT consists in the evaluation of a Laplacian

$$\sigma = \frac{\nabla^2 \varphi^{\pm}}{2\omega\mu_0}$$



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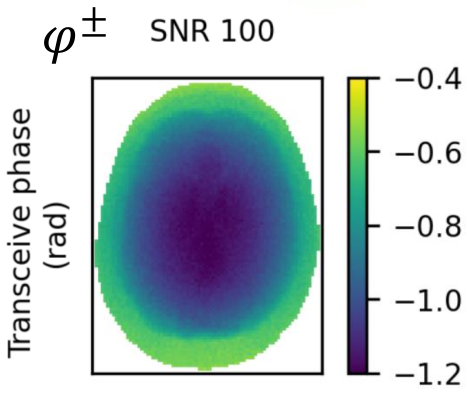

- The Laplacian is evaluated through the Savitzky–Golay filter:

  1. A moving kernel is centered at the pixel of interest
  2. The phase map is locally approximated with a polynomial fitting
  3. The Laplacian of the polynomial is evaluated analytically

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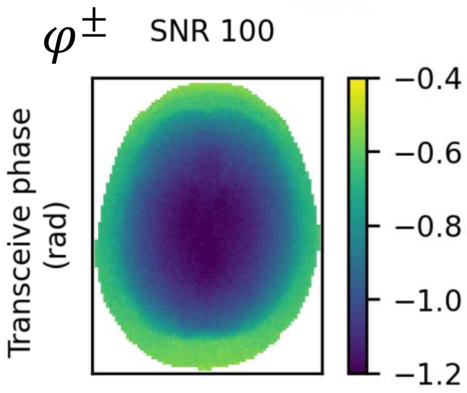

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$$\boldsymbol{x}^* = \underset{\boldsymbol{x}\in\mathbb{R}^n}{\operatorname{argmin}} \|A\boldsymbol{x} - \boldsymbol{b}\|^2 , \qquad A \in \mathbb{R}^{m,n}, \boldsymbol{b} \in \mathbb{R}^m$$

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$$\boldsymbol{x}^* = \underset{\boldsymbol{x}\in\mathbb{R}^n}{\operatorname{argmin}} \|A\boldsymbol{x} - \boldsymbol{b}\|^2$$

- The QR decomposition of A allows to:
    1. Solve the problem
    2. Estimate the uncertainty of $\boldsymbol{x}^*$

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- $Q \in \mathbb{R}^{m,m}$ is an orthogonal matrix
- $R \in \mathbb{R}^{m,n}$ is an upper triangular matrix, $R = \begin{pmatrix} \tilde{R} \\ O \end{pmatrix}$
- $A = QR$

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$$\|A\boldsymbol{x} - \boldsymbol{b}\|^2 = \|R\boldsymbol{x} - \underbrace{Q^{\mathrm{T}}\boldsymbol{b}}_{\boldsymbol{c}}\|^2 = \left\|\begin{pmatrix} \tilde{R}\boldsymbol{x} - \boldsymbol{c}_1 \\ \boldsymbol{c}_2 \end{pmatrix}\right\|^2 = \|\tilde{R}\boldsymbol{x} - \boldsymbol{c}_1\|^2 + \|\boldsymbol{c}_2\|^2$$

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Method

$$x^* = \underset{x \in \mathbb{R}^n}{\operatorname{argmin}} \|Ax - b\|^2$$

- The QR decomposition of A allows to:
  1. Solve the problem
  2. Estimate the uncertainty of x^*

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$$\|Ax - b\|^2 = \|Rx - \underbrace{Q^{\mathrm{T}} b}_{c}\|^2 = \left\| \begin{pmatrix} \tilde{R}x - c_1 \\ c_2 \end{pmatrix} \right\|^2 = \|\tilde{R}x - c_1\|^2 + \|c_2\|^2$$

$$\boxed{x^* = \tilde{R}^{-1} c_1}$$

$$\boxed{\chi_n^2 = \frac{\|c_2\|^2}{m - n}}$$

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Method

- χ_n^2 estimates the variance of $\boldsymbol{b}$ components, assumed to be independent identically distributed

$$\Sigma(\boldsymbol{b}) = \chi_n^2 I \quad \Longrightarrow \quad \Sigma(\boldsymbol{c}) = Q^{\mathrm{T}}\Sigma(\boldsymbol{b})Q = \chi_n^2\left(Q^{\mathrm{T}}Q\right) = \chi_n^2 I$$

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- The law of propagation of the uncertainty can be used again

$$\Sigma(\boldsymbol{x}^*) = \tilde{R}^{-1}\Sigma(\boldsymbol{c}_1)\tilde{R}^{-\mathrm{T}} = \chi_n^2\tilde{R}^{-1}\tilde{R}^{-\mathrm{T}}$$

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- Finally, the Laplacian is computed as a linear combination of the polynomial coefficients

$$\sigma = \boldsymbol{w}^{\mathrm{T}} \boldsymbol{x}^* \quad \Longrightarrow \quad u(\sigma)^2 = \boldsymbol{w}^{\mathrm{T}} \Sigma(\boldsymbol{x}^*) \boldsymbol{w} = \chi_n^2 \left(\tilde{R}^{-\mathrm{T}} \boldsymbol{w}\right)^{\mathrm{T}} \left(\tilde{R}^{-\mathrm{T}} \boldsymbol{w}\right) = \chi_n^2 \left\| \tilde{R}^{-\mathrm{T}} \boldsymbol{w} \right\|^2$$

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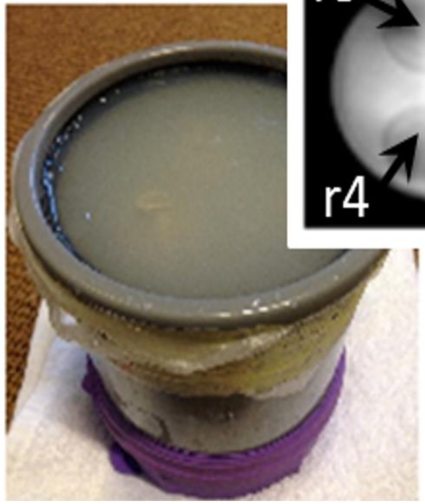
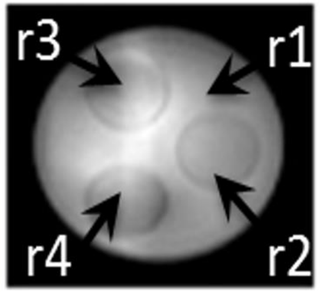
Application to phantom measurements

- $\varphi^{\pm}$ acquired by a 3 T MRI scanner (Ingenia, Philips HealthCare, Best, The Netherlands)
  - Body coil in transmit mode
  - 15-channel head coil in receive mode





Region	Radius [cm]	σ [S/m]
r1	6.25	0.1
r2	2	1
r3	2	2
r4	2	4

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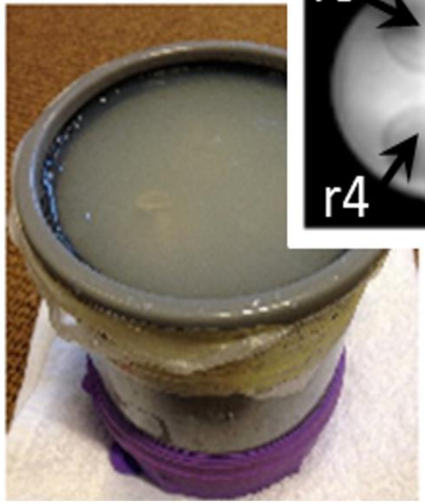
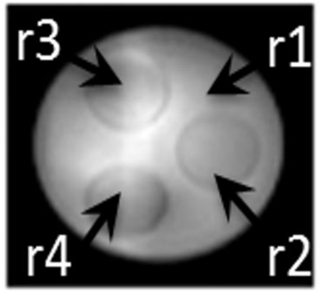
- Two T1-weighted Spin-Echo maps acquired with opposite readout polarities and combined for the $\varphi^{\pm}$ acquisition
  - TR = 900 ms; TE = 5 ms; resolution = 2 mm isotropic





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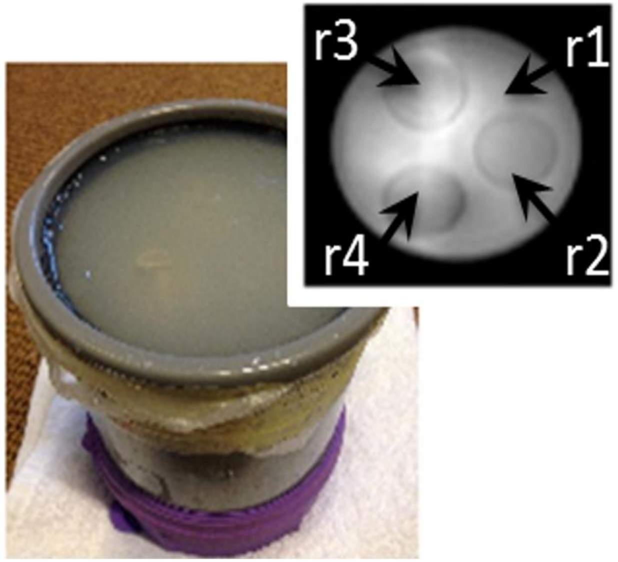
- Agar-based (2 %) phantom with inner compartments with NaCl added.



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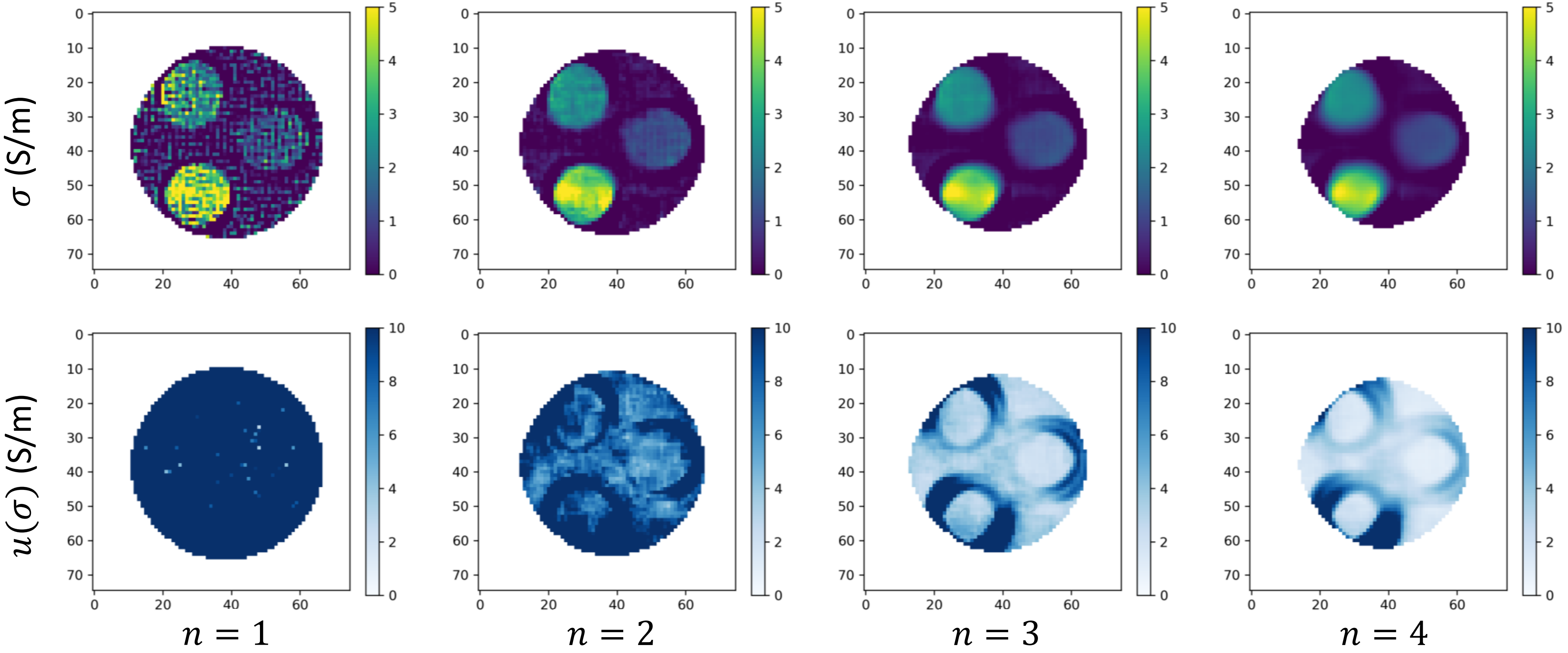
Application to phantom measurements

- The kernels of the Savitzky-Golay filter are square of $2n + 1$ pixels per edge
- Large uncertainties at compartment boundaries ➔ Model errors are included



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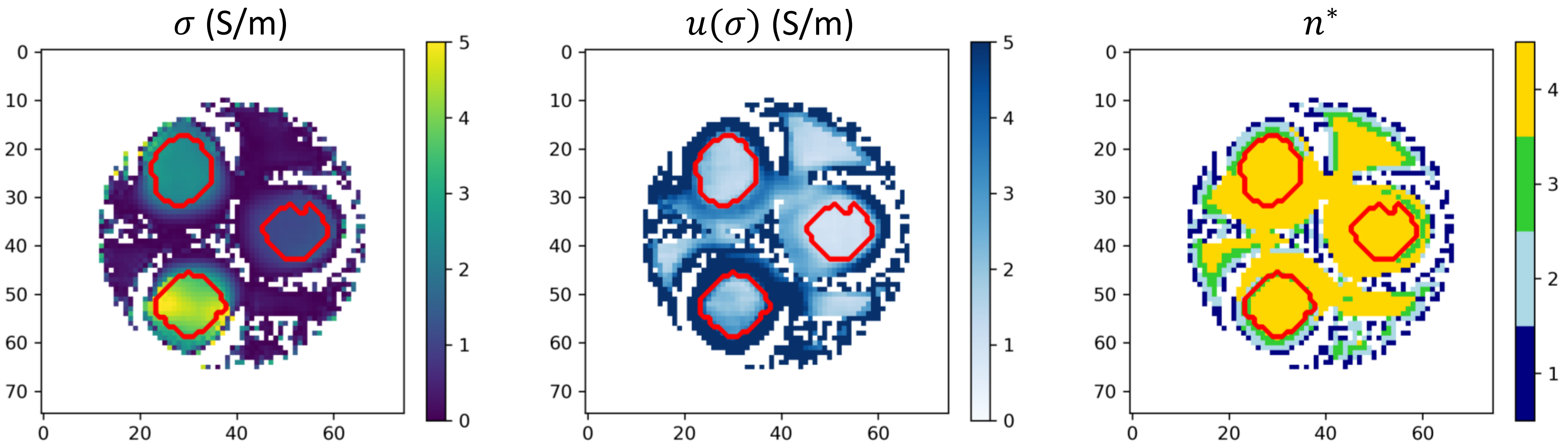
Application to phantom measurements

- The maps can be compared and combined pixel-by-pixel looking for:
  - Minimum uncertainty
  - Positive conductivity (physical constraint)

- Highlighted the regions where the relative uncertainty $u(\sigma)/\sigma$ is below 1



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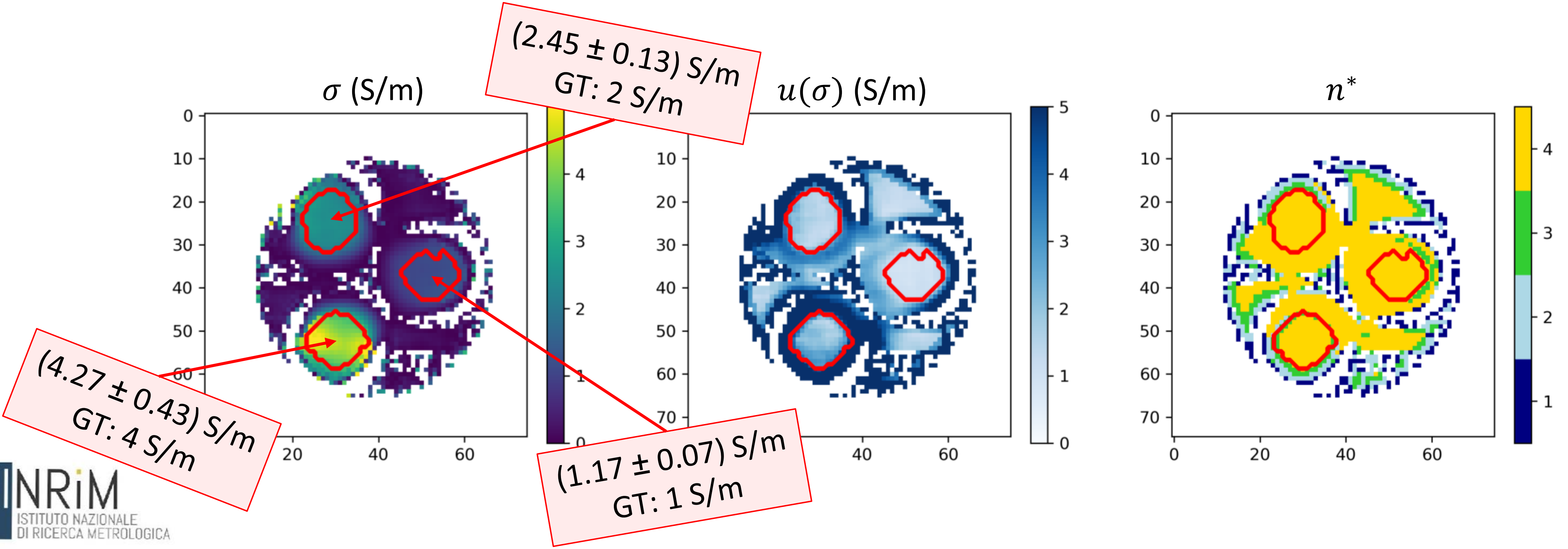

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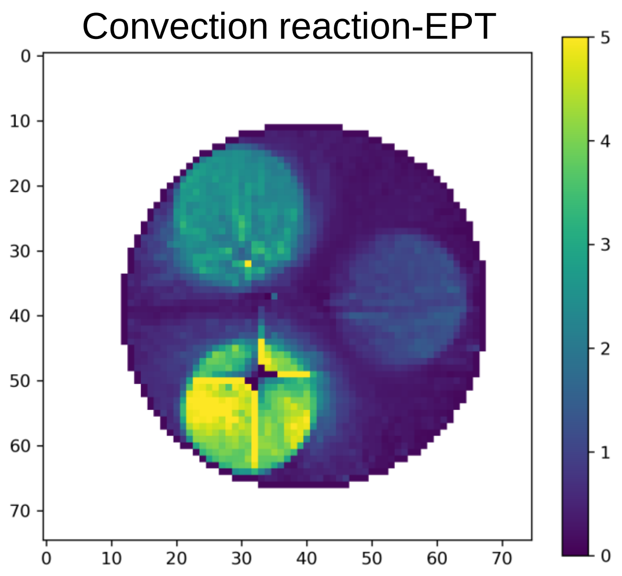
Hybridization of EPT techniques

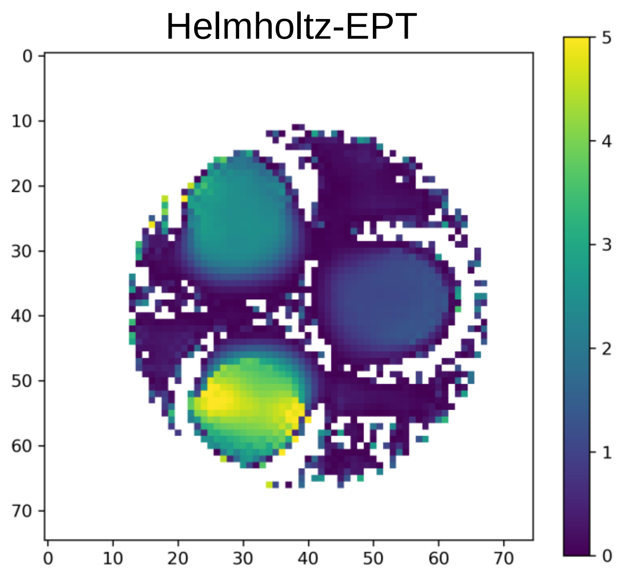
- Phase-based convection reaction-EPT is a technique based on a PDE

$$\nabla \cdot (\rho \nabla \varphi^{\pm}) = 2\omega\mu_0$$

- Once discretized, it can be written as a least squares problem

- The problem can be regularized with a weighted additive term derived by the Helmholtz-EPT solution





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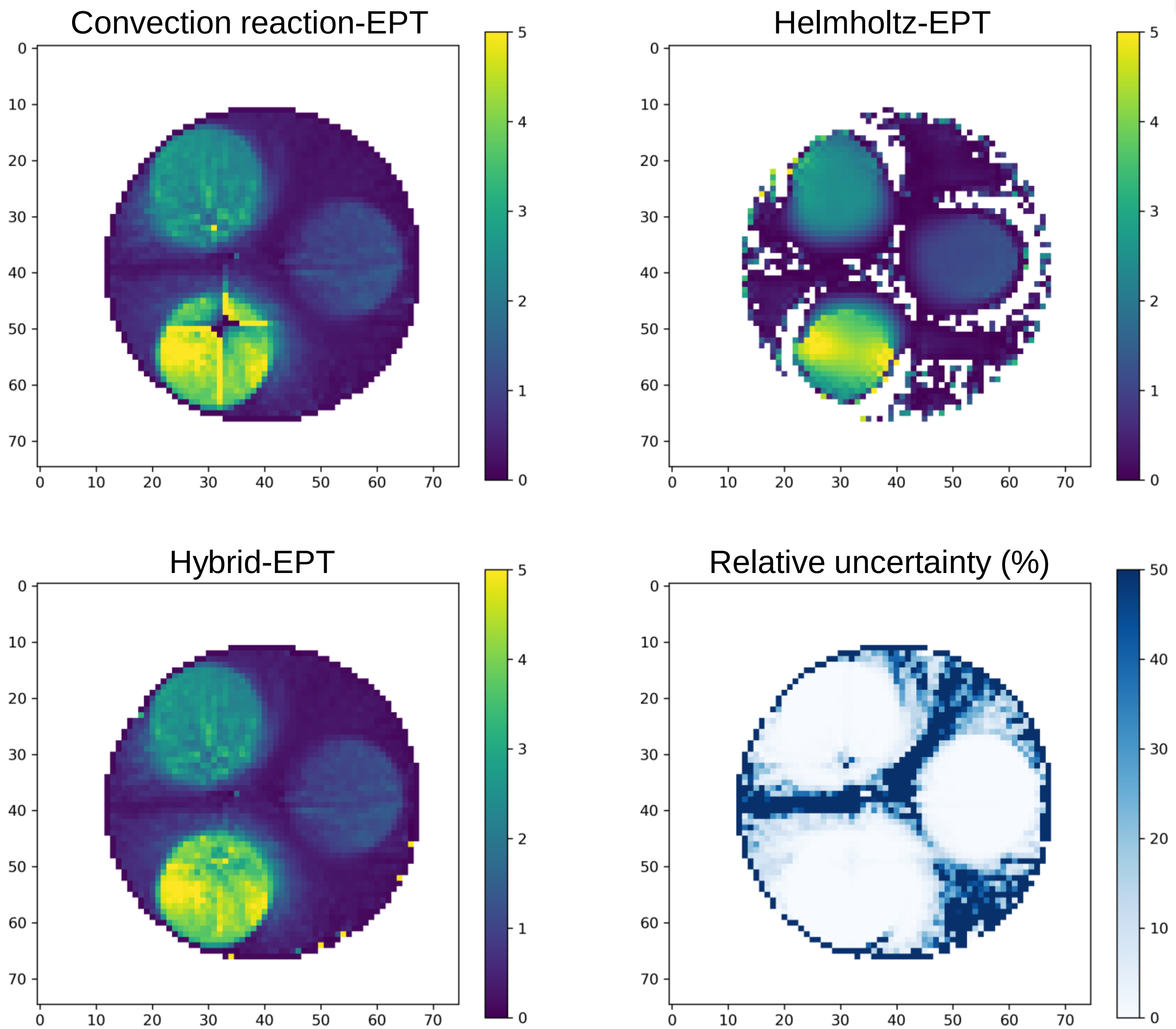
- Using the QR decomposition the problem can be solved obtaining also the uncertainty map.



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Conclusions

- A pixel-wise estimation of the uncertainty in Helmholtz-EPT has been presented
- The procedure can be applied also to other techniques based on the least squares problem
- Such an uncertainty estimation is fundamental for:
  - Quantitative pixel-wise comparison
  - Weighted hybridization of EPT techniques

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Thank you for your attention!

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