

# KALMAN filtering to extract patterns and metrological data from dynamic flowmeter calibrations

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# Overview

1. Introduction: CETIAT, « Dynamic calibration » and its meaning in the scope of liquid flow metrology
2. Kalman filtering, and its adaptation to dynamic mass flow rate calculation
3. Pattern recognition in dynamic liquid flow profile measurements

# Summary

1. Introduction: CETIAT, « Dynamic calibration » and its meaning in the scope of liquid flow
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# Introduction - CETIAT primary standards for liquid flow

- CETIAT is the French Designated Institute by

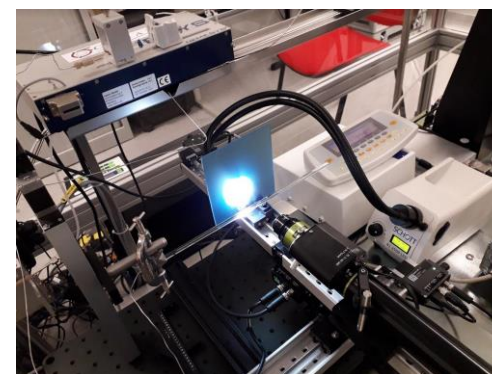


- Primary standards at CETIAT for liquid flow:

- 1 g.h<sup>-1</sup> to 100 kg.h<sup>-1</sup> (micro-flow)
- 5 kg.h<sup>-1</sup> to 50 000 kg.h<sup>-1</sup> (macro-flow)
- 12 °C to 90 °C (water temperature)
- Gravimetric method (ISO4185)
- U = 0.05 % (k=2)
- ISO17025 (COFRAC) accredited
- Accredited for calibration methods development:
  - Dynamic flow calibration (see next slides)
  - Nano-flow ( 10 µg/h to 1 g/h)



*CETIAT, liquid flow laboratory, 1978 (left) and 2022 (right)*



*CETIAT, micro (left) and nano (right)-flow standards, 2022*

# Introduction – dynamic flow calibration




- Two interpretations of « dynamic » calibration coexist:
  - In the sense of the measurement process for example, dynamic weighing in gravimetric methods
  - In the sense of the flow properties fluctuations (flow rate, pressure, temperature)
- Dynamic Calibration methods for liquid flowmeters have been extensively presented and discussed in literature
  - FLOMEKO, ISFFM, publications from NIST, PTB, ...
  - See last slide for full list of references

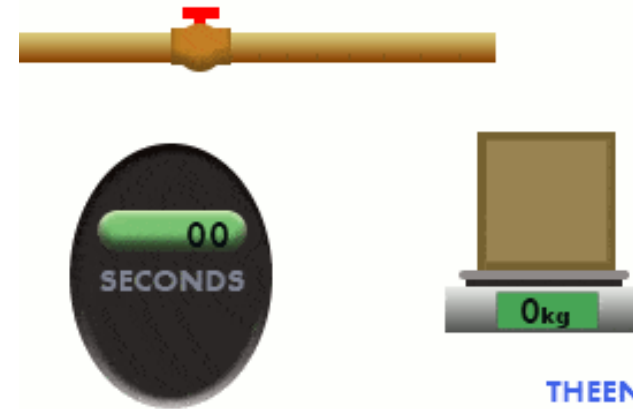


# Introduction – dynamic flow calibration

## Gravimetric method

**ISO 4185:** Measurement of liquid flow in closed conduits — Weighing method

Reference flow rate  
calculated   $Q_{REF} = \frac{\Delta m}{\Delta t}$   Weighted mass  
 Measured time



**Relative measurement error [%] of flow meter under calibration (DUT):**  $E_i = \frac{Q_i^{DUT} - Q_i^{REF}}{Q_i^{REF}}$

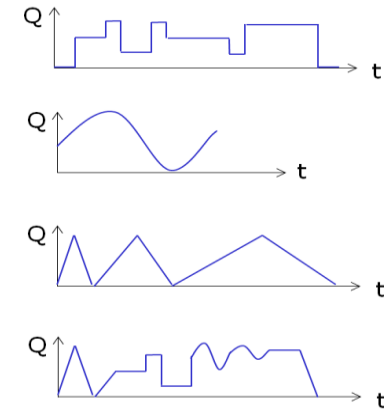
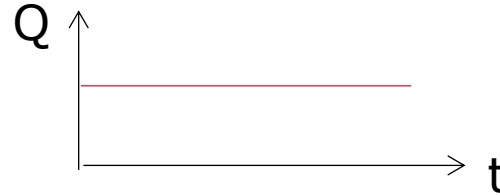
With:

$Q_i^{DUT}$  flow rate measured by device under test (DUT, flow meter under calibration).

$Q_i^{REF}$  reference flow rate measured by the primary standard test bench.

# Introduction – dynamic flow calibration

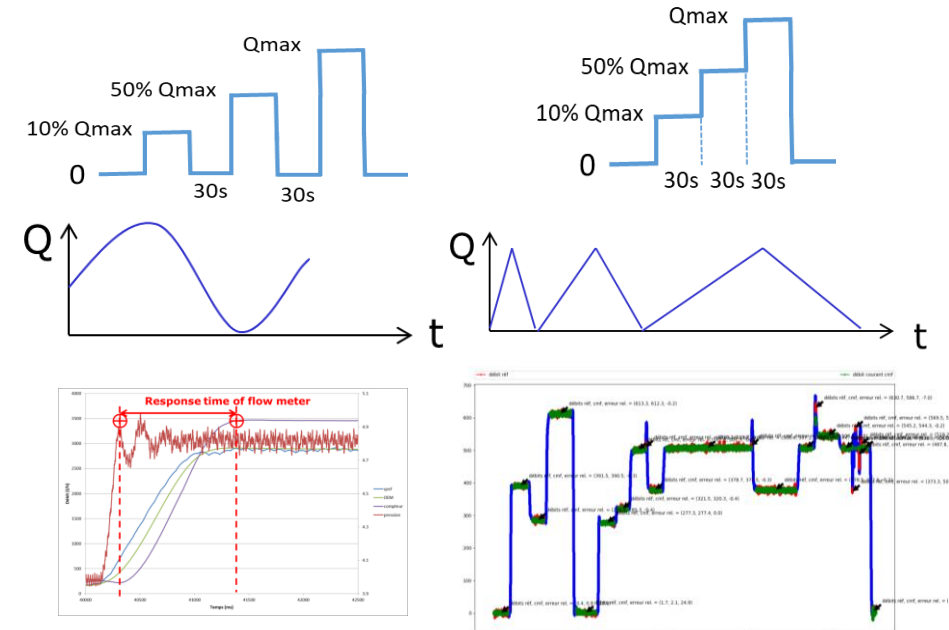
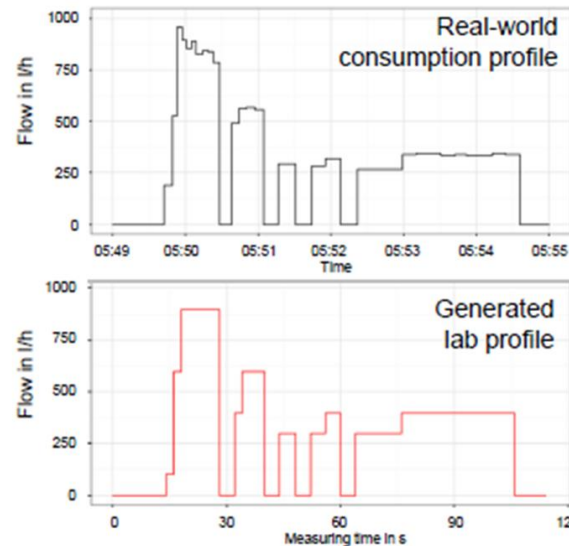
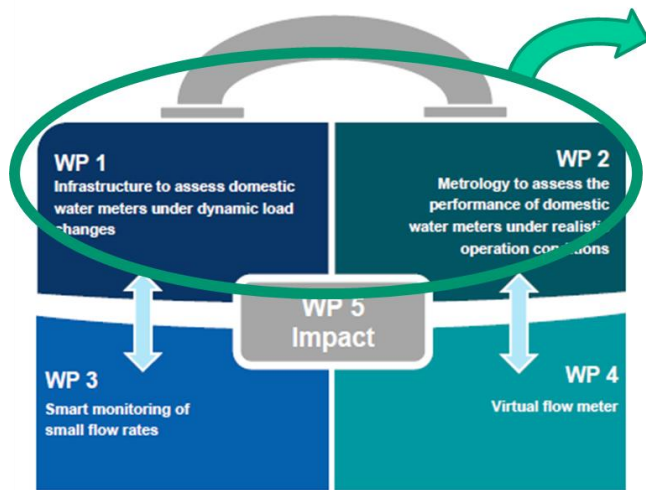
- Static vs. dynamic calibration methods in liquid flow metrology:



	Static	Dynamic
<b>Flow rate</b>	Constant	Constant or fluctuating (steps, ramps, oscillations...)
<b>Calibration time</b>	10 min. to several hours	1 minute to a few 10 min.
<b>Parameters evaluated during calibration</b>	Average measurement error Repeatability Reproducibility	<b>Dynamic response</b> of the DUT: Instantaneous (dynamic) error, Relative stability Response time +Repeatability+Reproducibility

# Introduction – dynamic flow calibration

## At CETIAT:



Metrology for Real-World  
Domestic Water Metering

**2018-2021**

**EMPIR**  

The EMPIR initiative is co-funded by the European Union's Horizon 2020 research and innovation programme and the EMPIR Participating States

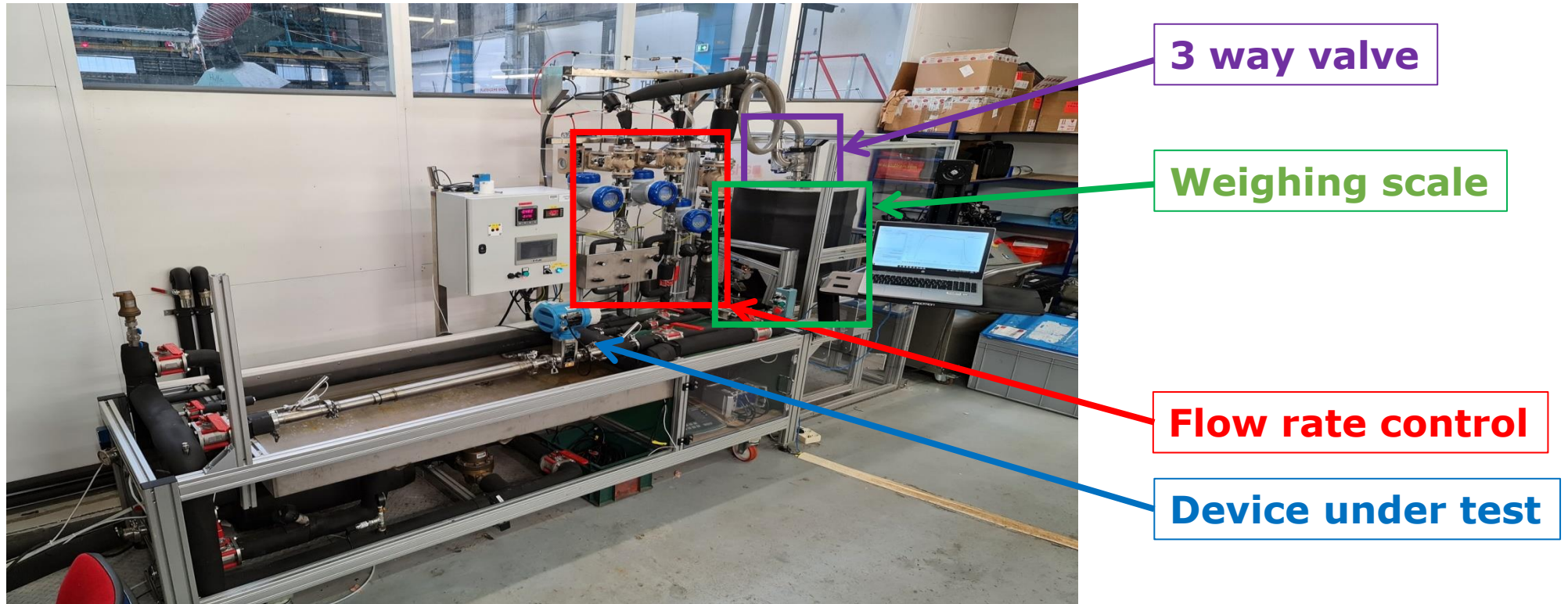
**JRP METROWAMET**

Goal: development and validation of primary standards for the calibration of flow and water meters under dynamic conditions



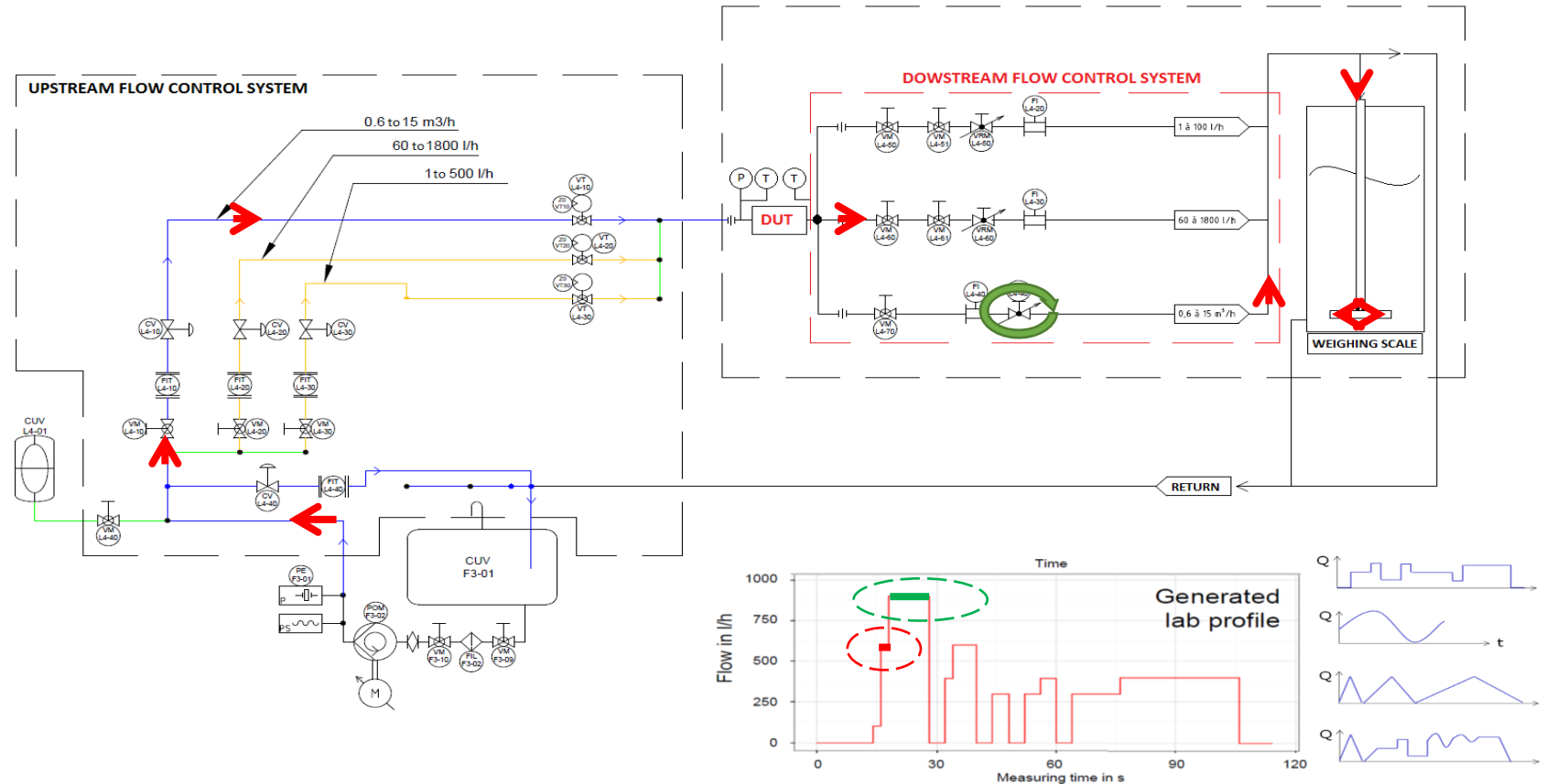
# Introduction – dynamic flow calibration

## CETIAT's dynamic test bench:

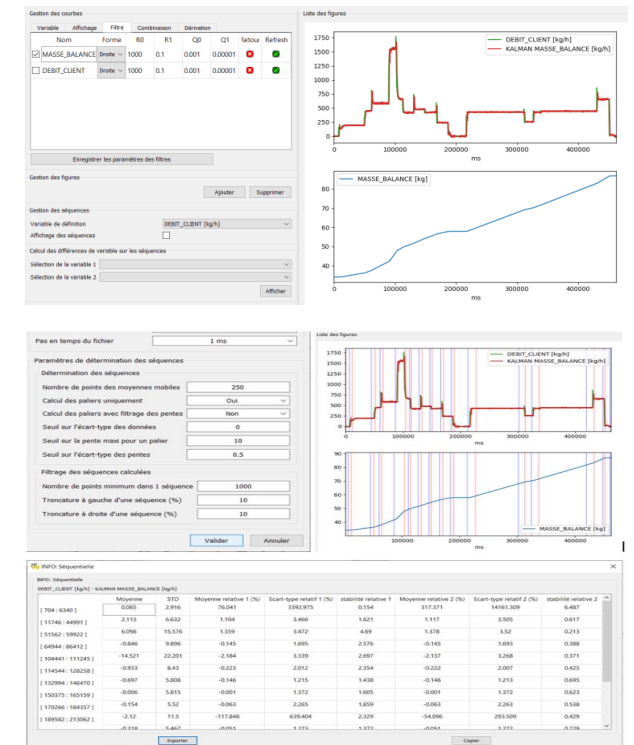


# Introduction – dynamic flow calibration

## Functional scheme of CETIAT's dynamic primary standard:



## Dedicated tool:



recording, visualisation,  
Filtering, pattern recognition and  
export of calibration results, ...

# Introduction – dynamic flow calibration

## Exemples of dynamic liquid flow calibration:

**Dynamic error :**  $E_{dyn} = \frac{1}{n} \sum_1^n Q_i^{DUT} - Q_i^{REF}$

With:

$Q_i^{DUT}$  the device under test flow rate at instant i,

$Q_i^{REF}$  the reference flow rate at instant i,

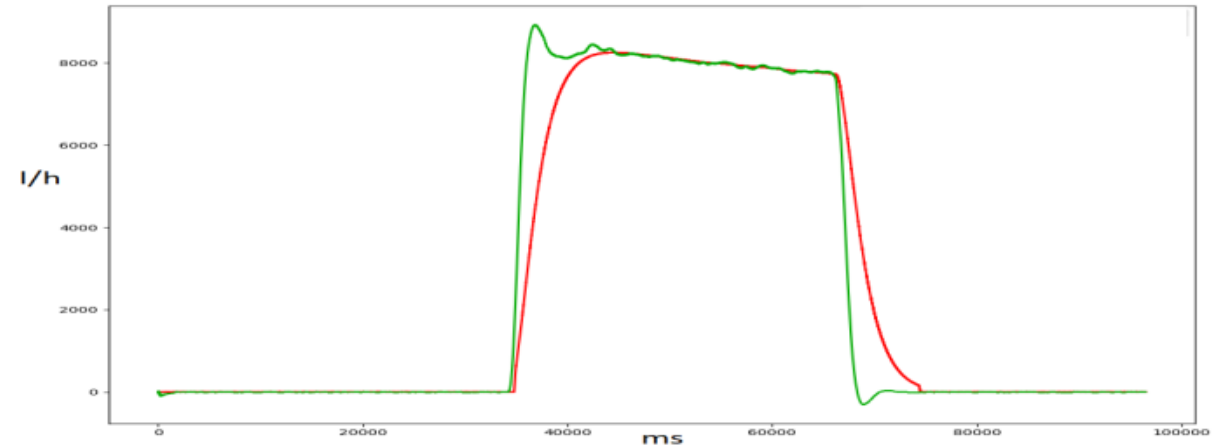
$n$  the number of flow samples on the chosen time interval.

**Relative stability:**  $Stab[\%] = \frac{\sigma_{DUT}}{\sigma_{REF}}$

With:

$\sigma_{DUT}$  the DUT's standard deviation on the chosen time interval,

$\sigma_{REF}$  the reference standard deviation on the chosen time interval.



Red: DUT's output, Green: reference flow rate

**Response time:** duration of the time interval between the instant of the step change of an input variable and the instant when the output variable reaches for the first time a specified percentage of the difference between the final and the initial steady-state value, as defined in entry 351-24-28 of IEC 60050-351:2006

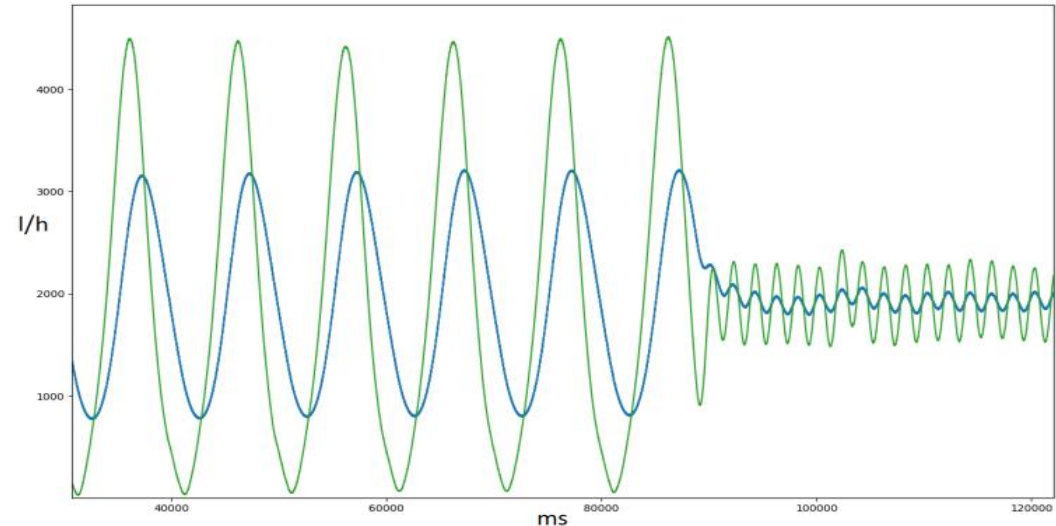
Parameter	Total step: 0 -> 8000 l/h -> 0	steady flow (8000 l/h)
relative error	-0.60%	-0.06%
relative stability	92.15%	102.08%
response time (s)	5	Not applicable

# Introduction – dynamic flow calibration

## Exemples of dynamic liquid flow calibration:

- Amplitude = 0 to 4500 l/h, frequency = 0.1 Hz, duration = 50 sec.
- Amplitude = 1500 to 2500 l/h, frequency = 0.5 Hz, duration = 30 sec

Results show the **effect of an increasing oscillation frequency on the DUT's metrological performances**: its relative error doesn't evolve, but its **relative stability decreases as the amplitude is dampened**, and the output signal phase shifts compared to the phase of the reference flow rate.



*Blue: flow rate measured by the Device Under Test*  
*Green: reference flow rate (primary standard)*

Parameter	0.1 Hz	0.5 Hz
relative error	-0.14%	0.09%
relative stability	55.83%	24.21%
Phase shift (deg)	36° (10% of 360°)	342° (95% of 360°)



# Summary

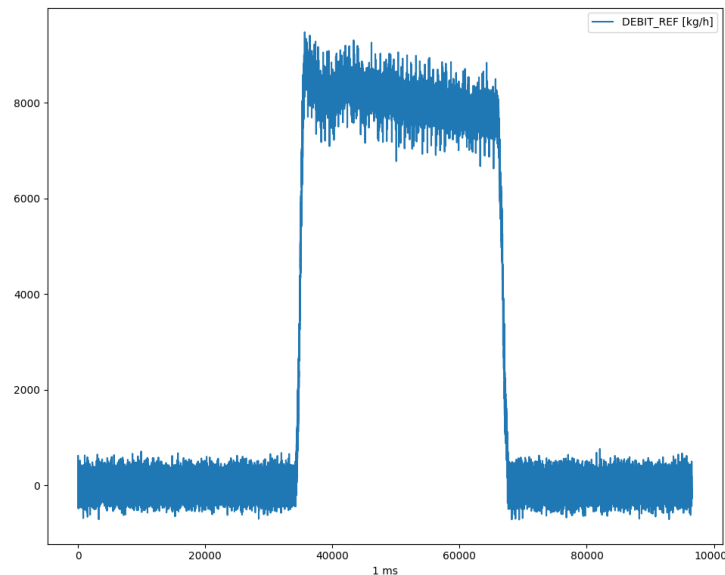
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# Kalman filtering

$$Q_{REF} = \frac{m_{t+\Delta t} - m_t}{\Delta t}$$

- Mass flow rate is calculated as the time-derivative of dynamic mass measurements acquired at 1 kHz sampling frequency, thus with a high level of noise:



*Blue: reference flow rate (primary standard)*

# Kalman filtering

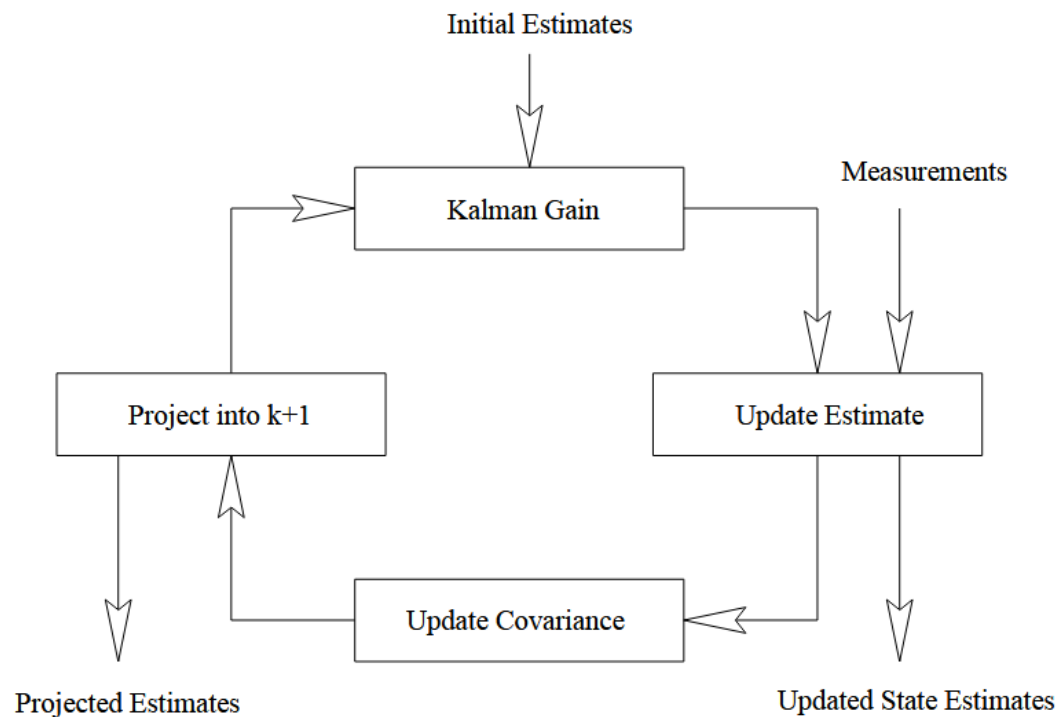
- To « denoise » (filter) the data, the solution consists in using a Kalman filter to smooth the mass measurement ( $x_0$ ) while simultaneously performing an estimation of the flow rate ( $x_1$ ). The state system is classically written as:

$$\begin{bmatrix} x_0^{k+1} \\ x_1^{k+1} \end{bmatrix} = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_0^k \\ x_1^k \end{bmatrix}$$

The above represent the model associated with a constant variation rate. To better perform, the Kalman filter needs a physical model as a predictor.

# Kalman filtering

## ➤ Kalman filter algorithm:



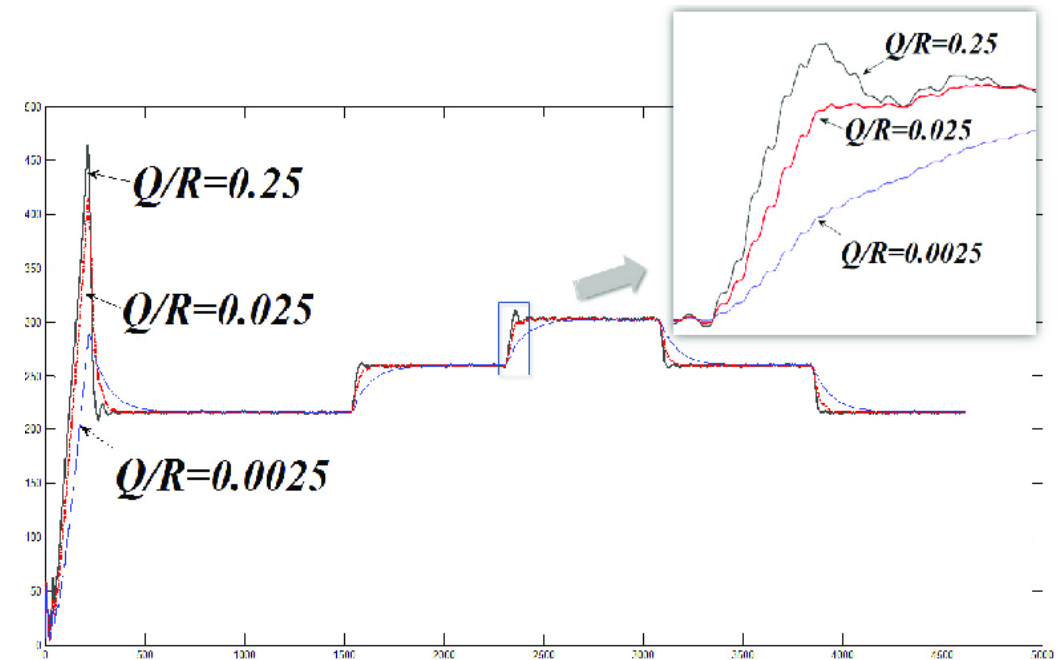
Description	Equation
Kalman Gain	$K_k = P'_k H^T (H P'_k H^T + R)^{-1}$
Update Estimate	$\hat{x}_k = \hat{x}'_k + K_k (z_k - H \hat{x}'_k)$
Update Covariance	$P_k = (I - K_k H) P'_k$
Project into $k + 1$	$\hat{x}'_{k+1} = \Phi \hat{x}_k$ $P_{k+1} = \Phi P_k \Phi^T + Q$

- **Q** is the covariance of the **process noise**. In our case, Q specifies how much the actual change of the mass deviates from the assumed flow model.
- **R** is the covariance matrix of the **measurement noise**, assumed to be Gaussian.

Source:  
<https://web.mit.edu/kirtley/kirtley/binlustuff/literature/control/Kalman%20filter.pdf>

# Kalman filtering

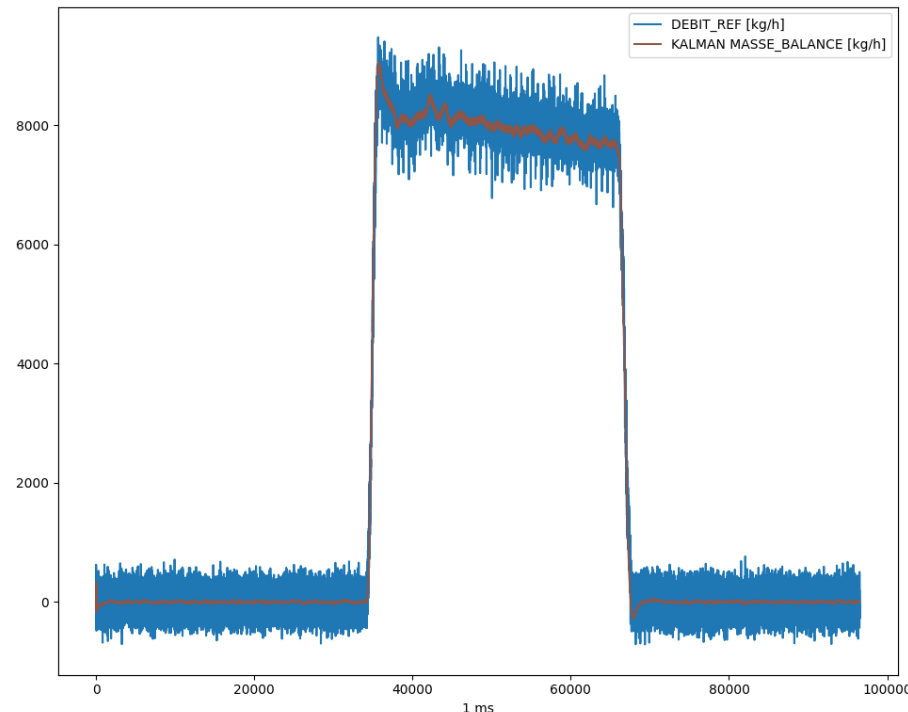
- Programming the classic Kalman filter algorithm is not a problem. The main difficulty consists in **optimizing** the coefficients of the matrices **R** of covariance and **Q** of the "noise" associated with the model.
- The **Q** process "noise" is **evaluated** from the measurements, and the R parameter must be **provided** by the user.
- In practice, it has been found that it is the **Q/R ratio** that must be taken into account, and this after an adjustment **balancing "noise attenuation" and "shape reconstruction"**. This is done **directly** in the algorithm.



# Kalman filtering

- Example of kalman filtered mass flow rate estimation with optimized parameters:

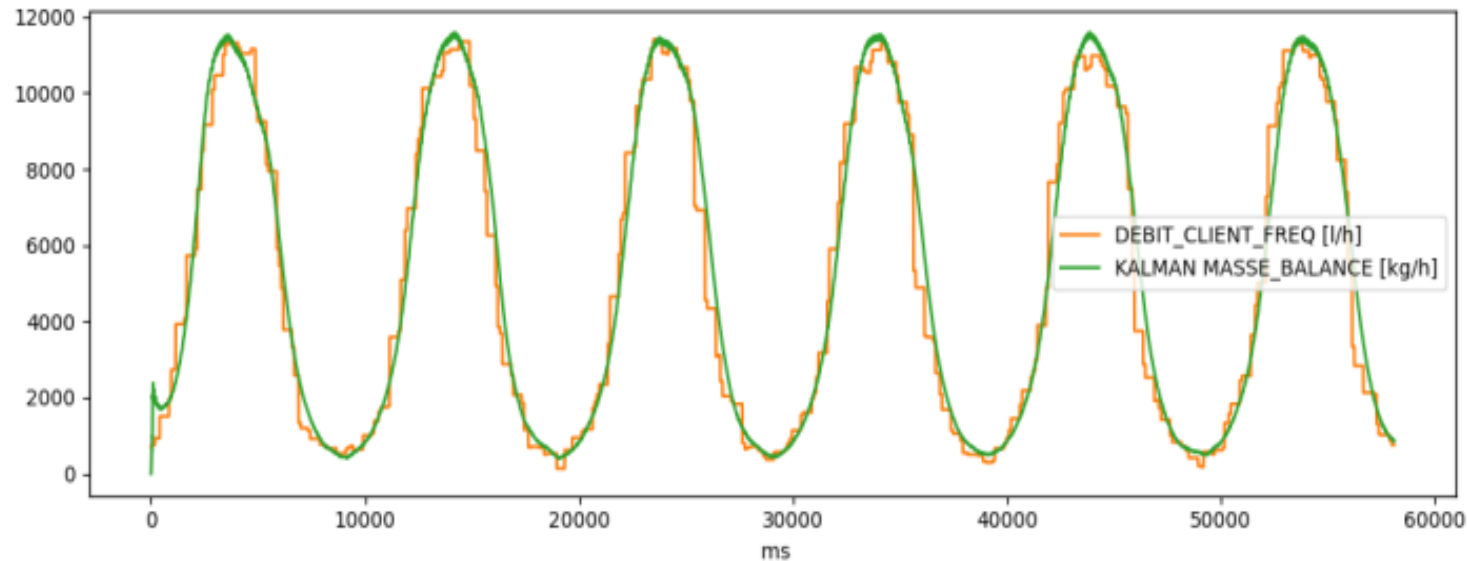
**The strength of the Kalman Filter lies in its ability to keep the dynamic of the signal while filtering it.**



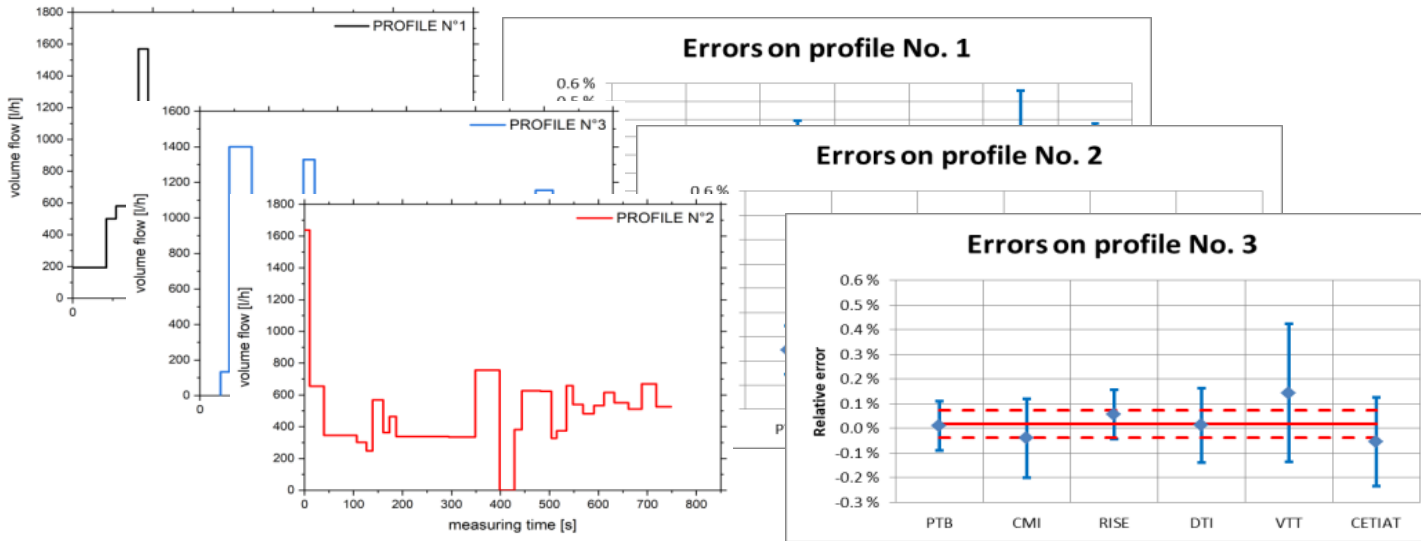


# Kalman filtering

- With an additional term in the state system matrix, the filter also works for **oscillating** flow rates (if the frequency is known).



# Kalman filtering – validation by ILC (EURAMET project 1506)

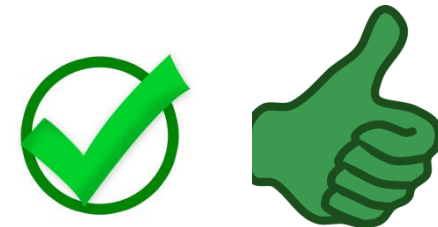


Uncertainty on the filtered dynamic reference flow rate:

$$U_{dynamic}(k=2) = 2 \sqrt{u_{static}^2 + u_{fit}^2}$$

With  $u_{fit}$  the mean of the sliding standard error over a one second time window (as a first approximation)

		CETIAT		PTB		FORCE		CMI		UME TUBITAK		RISE		DTI		VTT	
		U(k=2)	En	U(k=2)	En	U(k=2)	En	U(k=2)	En	U(k=2)	En	U(k=2)	En	U(k=2)	En	U(k=2)	En
PROFILE	1	0.20 %	0.51	0.10 %	1.04			0.16 %	0.03	0.33 %	0.04	0.10 %	0.42	0.11 %	0.29	0.40 %	0.23
	2	0.10 %	0.19	0.10 %	0.93	0.10 %	0.06	0.22 %	0.18			0.10 %	0.50			0.28 %	0.23
	3	0.18 %	0.42	0.10 %	0.10			0.16 %	0.38			0.10 %	0.45	0.15 %	0.04	0.28 %	0.46



**Dynamic primary standard for dynamic flow:  $U = 0.2 \%$  ( $k=2$ )**

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# Pattern recognition

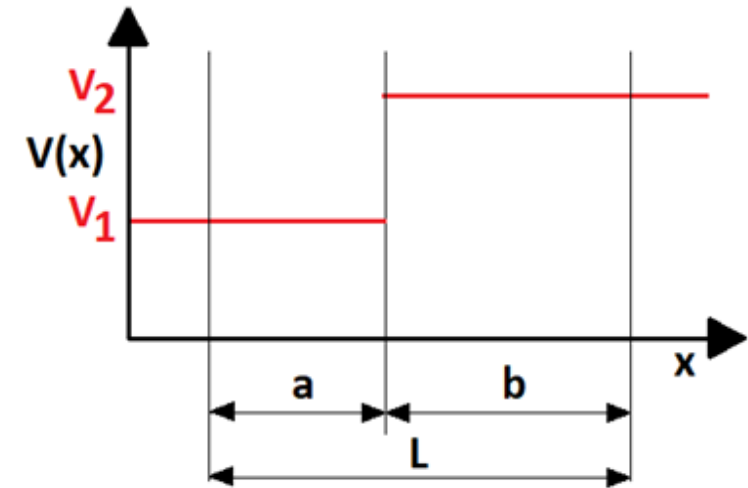
- After obtaining a series of flow values, resulting from the filtered measurements, it is necessary to extract the information corresponding to the flow rates. In the case of a row of flow steps, it is first necessary to **extract the temporal sequences corresponding to each step**, and finally to calculate the calibration-related informations.



Dynamic mass (input measurement)

# Pattern recognition

- The principle of the extraction algorithm is to achieve a **moving average of the standard deviations** on the estimate of the flow, and this over a range of measurements of constant length. The evaluation of the mean  $\sigma$  with respect to a threshold  $\sigma_{MAX}$  makes it possible to **determine the start and the end** of a pattern.
- The threshold  $\sigma_{MAX}$  depends on measured values and has currently to be provided by the user.



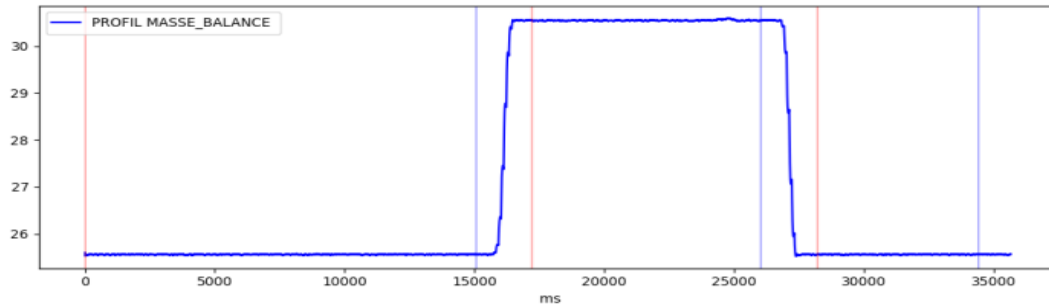
$$\sigma^2(X) = \frac{1}{L} \cdot \int_X^{X+L} (V(x) - \bar{V})^2 \cdot dx$$

$$\sigma_{MAX} = \frac{1}{2} \cdot |V_2 - V_1|$$

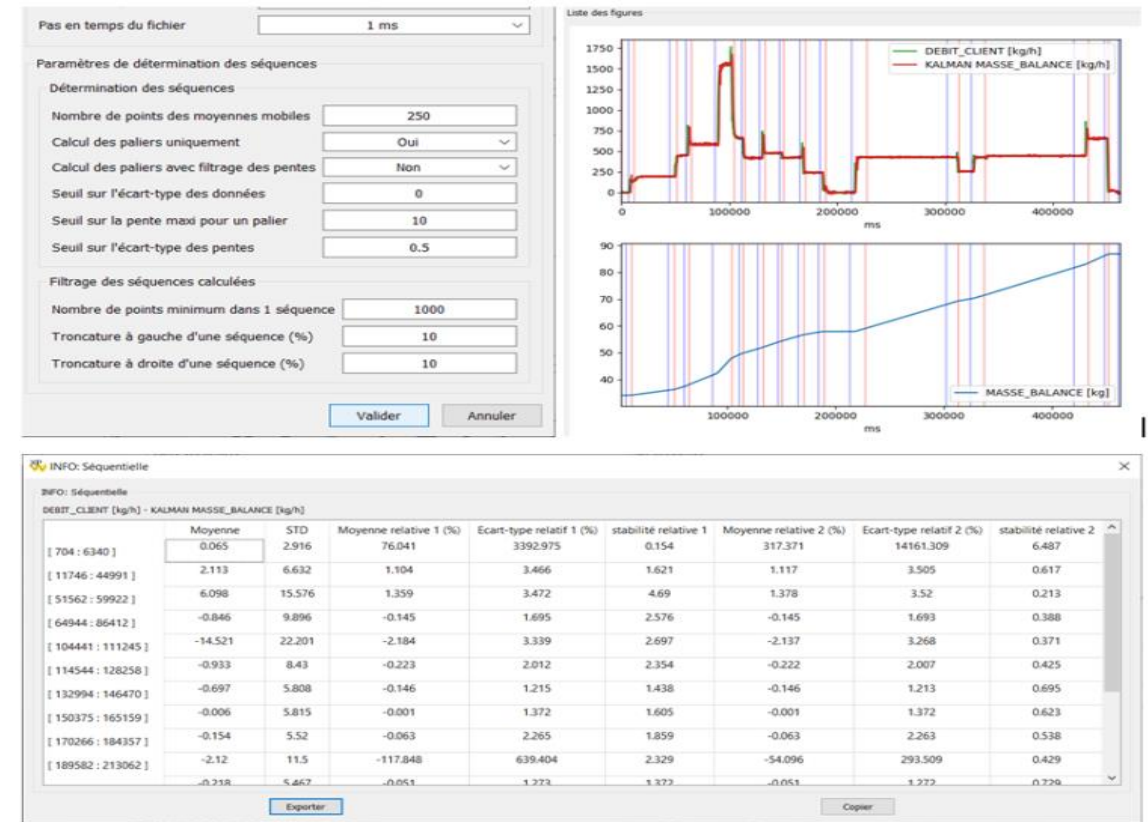


# Pattern recognition

- This algorithm naturally applies to a succession of steps, and for the case of **ramps**, the algorithm has to be applied to the **average of the slopes** of the estimate
- The process time is 10 s for a 100 s profile, which allow a **real time** use of the tool.



Example of recognition of a basic "step" pattern on a flow estimation by Kalman filtering (red/blue vertical lines -> start / end of a pattern)



Example of recognition of "steps" patterns on a real flow estimation by Kalman filtering

# Conclusion

- These 2 algorithms, **Kalman filtering** and **pattern recognition**, provide CETIAT with an **automatic, fast** and **accurate** tool for processing measurements from CETIAT liquid flow meter bench: the EXPERT tool [1] works not only in batch, but also in real-time for the acquisition and post-processing of flow measurements.
- The **EXPERT software** completes CETIAT capabilities in the proposal of services in the field of flow metrology.
- And if currently the algorithms at the base of the tool only apply to flows involving a linear state model, a future generalization to any type of flow (and other quantities) is possible by the use of an **extended Kalman filter** (EKF).
- This tool could also be applied to any other measurement with a more complex dynamic behavior, for use in **digital twin** or **process control** for example.

**[1] F. Ogheard, J. Noël, P. Granger, C.-A. Gassette, Flow Measurement and Instrumentation 84 (2022).**

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