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Determination of the minimum number of repeated observations at measurement procedure developing

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Abstract

In testing and calibration laboratories accredited for compliance with the requirements of ISO 17025:2017 [1] on the basis of paragraph 7.2.1.4 of this standard: “Laboratory-developed or modified methods can also be used”. When developing the tests or calibrations measurement procedure, one of the important issues is the choice of the minimum required number of measurements that provide, on the one hand, a given expanded measurement uncertainty, and, on the other hand, the minimum laboriousness of their implementation. It is generally accepted that the number of multiple measurements should be at least ten. This postulate is based on the document [2], in the Warning to paragraph 3.2.2 which states: “Generally, when the number n of repeated measurements is low (n <10), the reliability of a Type A evaluation of standard uncertainty has to be considered. If the number of observations cannot be increased, other means of evaluating the standard uncertainty have to be considered”. In fact, when carrying out repeated measurements, there is often no variability in the readings of the measuring device (for example, when calibrating a caliper with a gauge block. In this case, there is no point in taking repeated measurements at all.

The aim of the paper is to evaluation the minimum required number of observations based on the known dispersion of indications and type B uncertainty.

1. Basic algorithm of measurement uncertainty evaluation [3]

Expanded measurement uncertainty is equal to $U(y)=t_p(\nu_{eff})\cdot u_c(y)$ (1), where $u_c(y)$ is the combined standard uncertainty; $t_p(\nu_{eff})$ is the Student’s coefficient for the level of confidence p; ν_{eff} is the effective number of degrees of freedom, determined for the case of direct multiple measurements by the formula:

$$\nu_{eff}=(n-1)\left[1+\frac{u_B^2(y)}{u_A^2(y)}\right]^2.$$

Because the $u_c(y)=\sqrt{u_A^2(y)+u_B^2(y)}$, then, expressing $u_A(y)$ as $u_A(y)=s/\sqrt{n}$, where s is the standard deviation of indications, determined previously by a large number of observations (n>10); and $s/u_B(y)=\alpha$, we obtain an expression for the expanded uncertainty in the form:

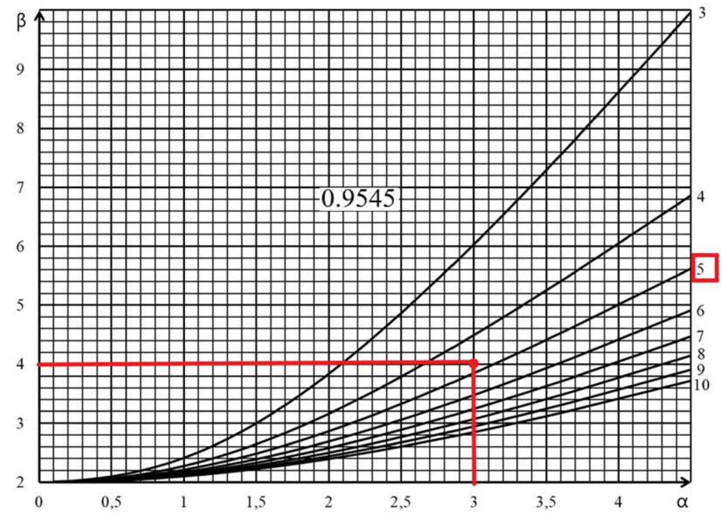
$$U(y)=t_p\left\{(n-1)\left[1+\frac{n}{\alpha^2}\right]^2\right\}u_B(y)\sqrt{1+\frac{\alpha^2}{n}}.$$

Thus, with known $u_B(y)$ and given $U(y)$, we obtain the dependence

$$\beta=\frac{U(y)}{u_B(y)}=t_p\left\{(n-1)\left[1+\frac{n}{\alpha^2}\right]^2\right\}\cdot\sqrt{1+\frac{\alpha^2}{n}},\qquad(2)$$

which, for a coverage probability of 0.9545, is shown in Fig.1.

Using dependence $\beta=\varphi(n,\alpha)$, it is possible to obtain the required number of observations for a given value $U(y)$ and known values of $u_B(y)$ and s. For this, it is necessary to find in Fig. 1 point of intersection of lines drawn perpendicular to the axes from the given values and β и α, and take the required number of observation results equal to n the value corresponding to the underlying curve closest to the point. For example, the point of intersection of perpendiculars for β=4 and α=3 will be corresponding to n=5.



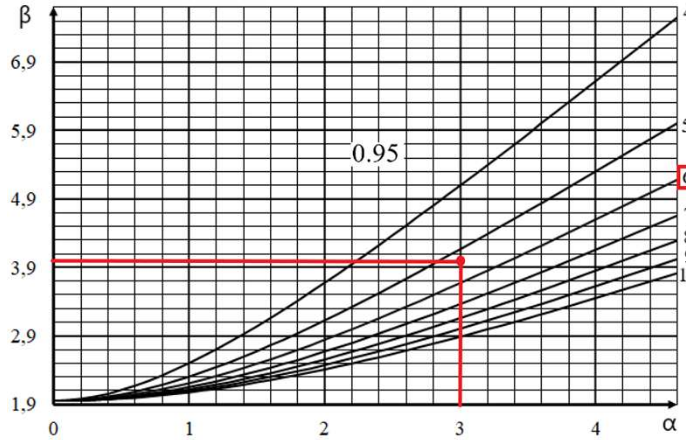
2. Monte Carlo method [4]

In this case, the computing model will look like $Y=\delta+\varepsilon$ (4), where δ is the correction for the instrumental error of the measuring instrument, which has a normal or uniform distribution law with zero mathematical expectation and a unit standard deviation; ε is the correction for a random measurement error having a t-distribution with zero mathematical expectation, ν=n-1 number of degrees of freedom and standard deviation:

$$s_t=\alpha\sqrt{\frac{n-1}{n(n-3)}}.\qquad(5)$$

The change of α was carried out in the range from 0.01 to 5. Using the NIST Uncertainty Machine program [5], for the given parameters α, n and u_B=1, the value of the expanded uncertainty U was determined for confidence levels of 0.95 after which the value β is determine. The dependences $\beta=\varphi(n,\alpha)$ are shown in fig. 2.

The algorithm for determining the required number of measurements is similar to that described in paragraph 1.



3. The Law of Expanded Uncertainty Propagation (LEUP)

A good approximation of the results obtained by the Monte Carlo method is the LEUP [6]. The bias of the values of the expanded uncertainty obtained by the LEUP from the values obtained by the MCM do not exceed 4.5%. For the measurement model (4), in this most case, the expanded measurement uncertainty will be equal to: $U(y)=\sqrt{U_A^2+U_B^2}$ (6), where U_A, U_B are the expanded uncertainties of type A and B, respectively, which are found by the formulas:

$$U_A=t_p(n-1)\cdot\frac{s}{\sqrt{n}}\quad(7);\qquad\qquad U_B=k_pu_B\quad(8).$$

In formula (7) $t_p(n-1)$ – Student’s coefficient for the confidence level p and the number of degrees of freedom n-1. In formula (8) k_p – coverage factor for normal or uniform distribution laws for confidence level p.

With this in mind, expression (6) can be rewritten as:

$$\beta=\frac{U(y)}{u_B(y)}=\sqrt{t_p^2(n-1)\frac{\alpha^2}{n}+k_p^2}.\qquad(9)$$

From here you can get:

$$\frac{\sqrt{n}}{t_p(n-1)}=\frac{\alpha}{\sqrt{\beta^2-k_p^2}}=\gamma.\qquad(10)$$

For the confidence level p=0.9545, the dependence $n(\gamma)$ is well approximated by the expression (the approximation error at the point n=3 is 2.85%, and for n>3 – no more than 1.5%): $n=4\gamma^2+2.5$ (11). For the confidence level p=0.95, the dependence $n(\gamma)$ is well approximated by the expression (approximation error at point a for n≥3 is not more than ±1.4%): $n=3.9\gamma^2+2.4$ (12).

If β=4 and α=3 and normal distribution assign to instrumental uncertainty tape B, for p=0.9545 we get k_p=2.0 γ=0.866 and n=5.5; and for p=0.95 we get k_p=1,96; γ=0.86 and n=5.3.

Conclusion

When developing testing or calibration measurement procedures, one of the important questions is the choice of the minimum required number of repeated observations n. The answer to this question will depend on the method used for measurement uncertainty evaluation. The report presents options for calculating n for the cases of applying the GUM uncertainty framework [3], the Monte Carlo method (MCM) [4], and the Law of expanded uncertainty propagation (LEUP) [6].

Using [3] and [4], the nomograms, that make it possible to determine n based on the given values of the expanded measurement uncertainty, on the standard deviation of the observed dispersion of indications, and on the standard instrumental uncertainty of type B, were obtained.

Using the LEUP [6], which makes it possible to get a good approximation of the results obtained by the MCM, the formulas, that allow determining n depending on the known characteristics of the observed dispersion of indications and the standard instrumental uncertainty of the type B for confidence levels of 0.95 and 0.9545, were obtained.

Comparisons of the received results were carried out and recommendations about their application are given.

References

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6. Zakharov I P, Botsyura O A “Estimation of expanded uncertainty in measurement when implementing a Bayesian approach”, Measurement Techniques, Vol. 61(4), pp. 342-346, 2018.